

Investigating the Onion Skin Method for Solving Simple Linear Equations

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Abstract

As one of the mathematical mnemonic tactics that use a visual approach, the *Onion Skin Method* (OSM) compares the structure of an onion to that of a mathematical problem. The study aimed to determine, through pre- and post-testing, the influence of incorporating the OSM into the learning of solving basic linear equations. The sample involved two Year 10 classes consisting of 31 students. Using a paired t-test, the study demonstrated a considerable improvement in students' ability to solve linear equations. A semi-structured interview with six students was done to examine their perceptions. Five themes were derived from the interviews: the time spent by students to solve simple linear equations, students' reactions to their performance on the post-test, students' preferred method for solving linear equations, students' perception of the OSM, and students' future use of the OSM. The findings indicated that the OSM accommodates the learning styles of the participants, which predominantly employ mnemonic strategies. This research may inspire teachers to use alternative methods when instructing students on solving fundamental linear equations, allowing them to modify their instruction or develop more practical lessons for teaching.

Keywords: Onion skin method, Algebra, Solving linear equation, Secondary Mathematics.

Introduction

Mathematics principles are deemed helpful in everyday life (Hagan et al., 2020; Nordin et al., 2017). Mathematics is regarded as one of the essential school courses as the majority of colleges and universities continue to use it as one of their admission or even employment requirements (Adnan et al., 2021; Farooq & Shah, 2008; Hagan et al., 2020; Matzin et al., 2013; Shahrill et al., 2013; Yahya & Shahrill, 2015). Numerous researchers consider algebra one of the most demanding topics, which might impact their numeracy skills (Adu et al., 2015; Jagnandan & Jagnandan, 2021; Johari et al., 2020; Jupri et al., 2014; Nordin et al., 2017). Many sub-topics rely on students' proficiency in using algebra to find the solution to any given equation (Ahmad & Shahrill, 2014; Jagnandan & Jagnandan, 2021; Sarwadi & Shahrill, 2014).

According to Adnan et al. (2021) and Grønmo (2018), algebra can be interpreted as a mathematical language. Therefore, it is no surprise that algebra can translate word problems mathematically and be used across any topical mathematical problems that need to be solved (Adnan et al., 2021; Grønmo, 2018; Linsell, 2009). Even though it may be necessary to address problems from other topics, most students worldwide, including those in Brunei Darussalam, have regarded algebra as one of the most challenging topics to learn (Adu et al., 2015). Although students typically excel at turning word problems into algebraic expressions or equations, it is equally possible that they will struggle to solve algebraic equations. Hypothetically, students were exposed to various methods for solving linear equations. Still, students make careless mistakes when solving a simple linear equation (Egodawatte, 2009; Sarwadi & Shahrill, 2014; Yahya & Shahrill, 2015), mainly when it involves more than one

step of solving. Therefore, educators must actively seek innovative teaching methods to improve students' numeracy levels (Ministry of Education, 2013).

A few strategies have been introduced to aid students' learning in solving algebraic equations (Damit et al., 2017). However, no concrete evidence exists on the best practical method to teach and learn the topic. Students in Brunei were exposed to various teaching and learning strategies for solving algebra as early as the upper elementary level when they were required to discover the unknown value in a basic linear equation. According to Damit and colleagues, students at this level must use the *guess-and-check* strategy. Children will use this method to anticipate the unknown number and determine whether both sides have the same weight of value (Linsell, 2009; Te Maro et al., 2008). Also, Linsell (2009) mentioned that many students in New Zealand employ this method to solve problems such as $b + 9 = 12$. This may appear ideal for students with lower cognitive abilities to guess any integer number as the unknown. In this situation, those students are primary school learners and have not been introduced to more complex problem-solving methods.

At the secondary level, however, the complexity of algebra gradually increases as students' cognitive abilities improve. The solution to the equation will include fractional values in addition to integer numbers. Therefore, the system for educating secondary school students must be revised. The *transposition method* is one of the well-known procedures (Damit et al., 2017). In the transposing approach, students must transpose or transfer a term by changing its sign from one side to the other to solve it, as shown in Figure 1. Conceptually, the transposition of a term will restore equality to both sides of an expression and directly simplify the linear equation (Anthony & Burgess, 2014; Hall, 2002; Nordin et al., 2017).

Linsell (2009) and Te Maro et al. (2008) also emphasised another technique known as *inverse operations* or the working backward method, which is comparable to the balancing method and is commonly used by secondary school teachers to teach the previously stated topic. Typically, this method was introduced to more adept students who can visualise simple facts or counting strategies. Students use this strategy when attempting two or more step equations (Figure 2). Students must balance each side by performing the same contrasting operation of the term intended to be eliminated on both sides until a solution-giving equation is obtained (Anthony & Burgess, 2014). Either way, the primary key for these methods to be successful is that students need to transpose or eliminate the term in the correct order, which needs to be noticed by the students more (Larino, 2018).

$$x - 3 = 5$$

$$\Rightarrow x = 5 + 3$$

$$x = 8$$

Figure 1. Transposing Method to Solve a Basic Linear Equation.

$$2x - 4 = 6$$

$$2x - 4 + 4 = 6 + 4$$

$$2x = 10$$

$$\frac{2x}{2} = \frac{10}{2}$$

$$x = 5$$

Figure 2. Balancing Method to Solve a Basic Linear Equation.

As one of the mathematical mnemonic tactics that use a visual approach, the *Onion Skin Method* (OSM) compares the structure of an onion to that of a mathematical problem. When

finding a solution to an algebraic equation, the students will be guided by the “onion skin” to visualise the sequential steps needed to balance and transpose terms. Ultimately, this study aims to look into the effectiveness of the OSM in supporting students in learning and solving algebraic problems so teachers can adapt their instruction or lesson planning to their students’ mathematical abilities and requirements. This study focused on a way to tackle the struggles faced by students in solving linear equations. A technique was used to reduce students’ errors when solving basic linear equations to give them a solid foundation in the subject matter.

The Onion Skin Method (OSM)

The OSM is an alternate procedural method that has been presented for solving simple linear equations. Yunus et al. (2016) have referred to the procedural teaching method as one of the most common teaching practices, whereby the teacher will constantly guide the necessary steps to answer the questions. The OSM was first discovered and reported by a teacher in a blog named *Passy’s World of Mathematics* (Passy, 2012). Passy outlined the three crucial steps to use the OSM in solving linear equations (Figure 3).

1. Draw the “skin”: Create the circles starting with the variable letter at the centre of the onion and continue to draw the layer of the skin to the subsequent closest terms from there. The outermost skin should enclose the entire equation.
2. “Peel” the onion skin: Work from the most significant outer skin and contrast the operations from the second outer skin until it reaches the variable letter centre.
3. Simplify the solution.

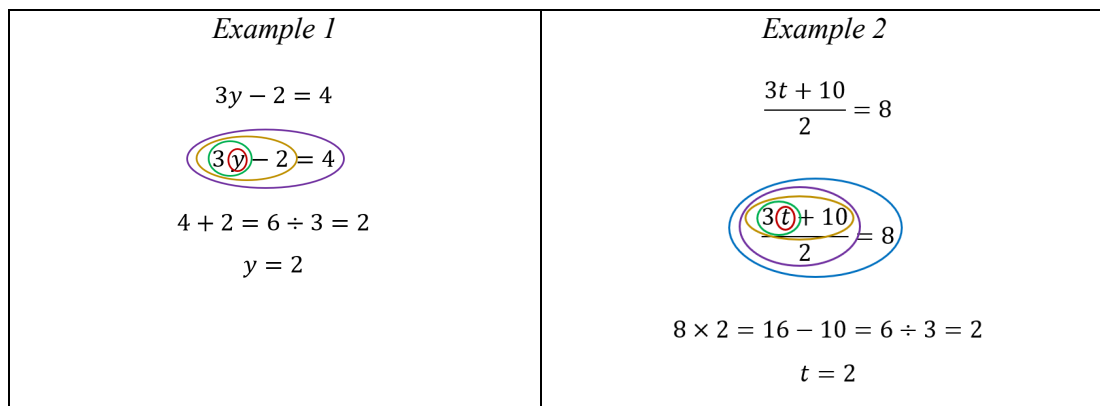


Figure 3. Examples of Applying the OSM in Solving Linear Equations.

Many experts feel that to develop an effective lesson plan, a teacher must be aware of the different learning styles in the class (Belecina & Ocampo, 2019; Coetsee & Kakoma, 2017; Egodawatte, 2009; Matzin et al., 2013; Othman et al., 2016; Te Maro et al., 2008; Shahrill et al., 2013). The learning styles relate to how students perceive the lesson and their competency to process information successfully (Pallapu, 2007; Reid, 1987). Therefore, understanding their style will support their academic success. Students employ various strategies to enhance their comprehension throughout the teaching and learning process. Research on the Index of Learning Style by Pallapu (2007) has identified that most students who participated in her survey were visual learners. This learning style will likely continue to be associated with more excellent academic success rates (Coetsee & Kakoma, 2017; Pallapu,

2007). The OSM can potentially aid visually oriented students in converting algebraic equations into visual representations.

Mathematical problem-solving has long been an area of interest for educators, philosophers, and cognitive scientists. Among the various techniques used to simplify algebraic expressions, OSM offers a structured, step-by-step approach, much like peeling away the layers of an onion. Although the OSM is not a formally recognised mathematical theory, it aligns with philosophical principles of reductionism, cognitive science research on learning, and practical applications across diverse disciplines.

The OSM represents a layered approach to problem-solving, a principle rooted in various philosophical traditions. One of the oldest philosophical systems that aligns with this strategy is reductionism, as René Descartes advocates. In his work “Discourse on Method”, Descartes argued that complex problems should be deconstructed into smaller, more manageable components before being solved (Grabiner, 1995). The OSM exemplifies this by focusing on the outermost operations of an algebraic equation before moving inward. Additionally, Jean Piaget’s constructivist theory supports the idea that understanding develops incrementally (Alanzi, 2016). The OSM encourages scaffolded learning, allowing students to grasp mathematical concepts in phases rather than attempting to solve the entire problem simultaneously. Similarly, the Cognitive Load Theory (CLT) suggests that students learn more effectively when information is presented in manageable segments (Paas et al., 2003; van Merriënboer & Sweller, 2005). The OSM aligns with this principle by reducing cognitive overload, enabling students to simplify one layer of an algebraic expression at a time. This approach enhances retention and reduces mental strain.

The effectiveness of the OSM may also be backed by research in mathematical cognition and educational psychology. Studies show that breaking information into smaller “chunks” enhances retention and recall (Gilchrist, 2015). The OSM improves understanding by treating the different components of an algebraic equation as distinct layers, which aids in cognitive processing. Lev Vygotsky’s Zone of Proximal Development (ZPD) concept indicates that learners achieve the best understanding when problems are divided into sequential phases that build on their existing knowledge (Margolis, 2020; Silalahi, 2019). The OSM functions as an effective scaffolding strategy, guiding students from basic to more complex procedures with structured support. Furthermore, research on mathematical cognition reveals that students who use systematic methods make fewer algebraic errors (De Bock et al., 2002). The OSM promotes a stepwise simplification approach, ensuring that each step is logically addressed before moving on. Thus, helps reduce errors. The OSM is particularly effective for solving linear equations requiring a series of structured steps. This approach uses a layer-by-layer simplification technique, ensuring that terms are correctly organised before simplification. The OSM is especially well-suited for linear equations involving parentheses, those needing several simplifications before isolating variables, and equations with fractions that require systematic elimination of denominators. However, for quadratic equations or those of higher order, additional methods, such as factoring or completing the square, may be necessary alongside the OSM to achieve a solution effectively.

Aside from Passy, (2012), the concept has been explored by various educational websites and online forums. For instance, Duncan Andrew (2017), characterises the onion method as a strategy for rearranging equations by “peeling away the layers” to isolate the desired variable.

These resources indicate that educators appreciate the OSM for its intuitive approach to teaching equation manipulation. However, due to a lack of empirical research, as far as is known, there is currently no evidence regarding how students perceive the OSM or its effectiveness compared to other instructional methods. Consequently, its impact on broader teaching strategies remains largely anecdotal. Given this context, the primary goal of the study would be to investigate the effectiveness of teaching students to solve linear algebraic equations using the OSM. This research could help fill a gap in the existing framework for future studies, particularly for researchers interested in applying the OSM to solve linear equations. The two research questions guiding this study are: 1) How does the OSM facilitate students in solving linear equations? 2) How do students perceive integrating the OSM in solving a simple linear equation?

Methodology

The study utilised qualitative and quantitative research methodologies, such as pre and post-tests and semi-structured interviews. Convenience sampling was employed to sample classrooms and students from the two Year 10 International General Certificate of Secondary Education (IGCSE) Mathematics classes, Aster and Buttercups, taught in the same single-gender government secondary schools in Brunei. There were 40 students enrolled in both classes; however, due to the cumulative absences of the other nine students over the research period, only the data from 31 students were evaluated in this study. The participants between the ages of 14 and 15 came from various academic backgrounds, with diverse capabilities ranging from “low” to “average”.

A group of students participated in a semi-structured interview during which a purposive sampling approach was employed. Judgemental or purposive sampling involves selecting participants based on the researchers’ judgement to focus on the students with specific characteristics who can contribute more effectively to a particular research (Etikan et al., 2016). In this sampling, the ‘representative’ sample was selected based on discretion or arbitrary criteria (Vehovar et al., 2016). Selecting participants according to defined performance criteria allows for a more systematic and insightful analysis of learning outcomes. The objective is to capture diverse learning experiences and gain a more comprehensive understanding of the impact of teaching OSM as an intervention on a diverse group of students by selecting six students for interviews based on their pre- and post-test results. The analysis is grounded in comparing the students’ pre- and post-test scores. Students were divided into groups based on their scores. Two students were chosen to represent the students who scored higher overall on the pre-test, two with better overall performance on the post-test, and two who maintained their performance level from the pre-test to the post-test. Classifying students into three different groups helps the researchers investigate several learning paths and evaluate the efficacy of the teaching strategy.

Additionally, a primary rationale for this selection is to guarantee the inclusion of participants with diverse learning styles in the study. By enrolling students who scored high on the pre-test, the researchers may see if those who already knew the content well benefited from the intervention. Similarly, students who fared better on the post-test with a lower pre-test score reveal how effective the strategy was in increasing conceptual understanding for initially

struggling learners. Finally, considering students whose performance remained constant helps us to determine if the intervention was neutral for some of them or whether other factors affected their capacity to develop. This grouping offers a balanced picture of several student experiences instead of a homogeneous or random sample (Alshenqeeti, 2014).

This selection approach also allows the researchers to understand individual learning variations. Not all students respond to a teaching method in the same manner, and comparing these three groups allows the study to investigate why some students progressed while others did not. This method looks at the cognitive and motivational elements of student learning and the simple analysis of test results (Alshenqeeti, 2014). Moreover, choosing this procedure enhances and strengthens the objectivity and validity of the research. Choosing students according to their performance trajectory instead of at random guarantees that the sample reflects various learners (Alshenqeeti, 2014). If the students were to be selected solely based on post-test results or randomly, the study might have been biased toward those who excel in mathematics, which limits the ability to generalise findings to a broader student population. Incorporating pre-test and post-test data mitigates potential bias and facilitates a more accurate evaluation of the efficacy of the teaching technique. The interview was subsequently examined using theme analysis.

The pre-test comprised ten questions on solving simple linear equations and rearranging algebraic equations, with each question having one mark for one-step solving and two marks for more than one-step solving and re-arrangement of formulae, totalling 19 marks. Any incorrect step or unanswered question resulted in a score of zero. The tests found in Table 1 were constructed and compiled concerning earlier IGCSE Mathematics examinations. The questions were divided into two sections: solving linear equations in the first section and rearranging algebraic equations in the second section. The questions on the pre-test were generally ordered by difficulty, beginning with the simplest one-step linear equation solution. The students were given 50 minutes to complete the test.

Table 1

Questions and the Descriptions in the Pre-test and Post-test

Section	Pre-test Number	Post-test Number	Question	Mark	Description
A	1	1	$\frac{x}{6} = 12$	1	One-step solving
	2	6	$3x - 4 = 14$	2	Two-steps solving
	3	7	$\frac{y + 1}{5} = 2$	2	Two-steps solving
	4	5	$\frac{p - 3}{5} = 3$	2	Two-steps solving
	5	2	$7x - 4 = 3x + 2$	2	Three-steps solving
	6	3	$6 - 2x = 3x$	2	Three-steps solving
	7	4	$7 - 3n = 11n + 2$	2	Three-steps solving
	8	8	$3(2z - 7) - 2(z - 3) = -9$	2	Three-steps solving, expansion
B	9	10	$c = 5a - 2b$	2	Rearranging algebraic equations
	10	9	$b = 3(a + 5)$	2	Rearranging algebraic equations

The pre- and post-tests are meant to be utterly comparable to one another. The post-test follows the same structure as the pre-test, containing questions, sections, and grading schemes. The order of the questions has been rearranged, resulting in a slight change in sequence (Table

1). Because of this, the items on the post-test were not arranged in a sequence that corresponded to their relative degrees of difficulty. Since there was no significant difference in the questions used in the pre- and post-tests, it is vital to point out that there is a chance of a limited memory impact, considering that both tests were carried out within a week.

Before conducting the study, both validity and reliability tests were performed. Two experts with more than five years of teaching experience in the Mathematics field validated the tests. In addition, the questions are commonly used in the actual IGCSE examinations, which can be considered valid tests. Each item's pre- and post-test score was either 0 for an incorrect response, 1 for a partially correct response, or 2 for a correct response. Utilising Cronbach's alpha, the instrument's internal consistency reliability was determined. Statistical software named Jeffrey's Amazing Statistics Program (JASP) was used to accomplish the calculation. A score of 0.7 on Cronbach's alpha scale (Table 2) indicates an adequate level of reliability (George & Mallery, 2003). It is generally agreed upon that the post-test questions were dependable because they were utilised in an environment conceptually comparable to the one used during the pre-test.

Table 2
Statistics on Frequentist Scale Reliability (Cronbach's Alpha Value)

Estimate	Cronbach's α
Point estimate	0.701
95% CI lower bound	0.392
95% CI upper bound	0.862

Results and Discussion

Descriptive Statistics of Pre-Test Results

Table 3 provides descriptive data for classes Aster and Buttercups' pre-test outcomes. Class Aster's mean score of 9.875 falls within the average mark range; however, class Buttercups' mean score of 3.933 may be considered low. Class Aster's lowest possible score is 4, and the most probable score is 17, whereas class Buttercups has a minimum possible score of 0 and a maximum score of 11 out of 19. Class Aster pre-test results have a standard deviation of 3.631, while the scores in class Buttercups have a standard deviation of 4.061. The aggregate mean value for both groups is 7.0, which falls inside the low mark range, and the standard deviation is 4.837. Although class Buttercups has a lower mean value than class Aster and both classes, the entire data sample has a higher standard deviation than individual classes.

Table 3
Pre-test Descriptive Statistics for Class Aster, Class Buttercups, and Overall Sample

	Class Aster	Class Buttercups	Overall Sample
Valid	16	15	31
Mean	9.875	3.933	7.000
Std. Deviation	3.631	4.061	4.837
P-value of Shapiro-Wilk	0.817	0.014	0.172
Minimum	4.000	0.000	0.000
Maximum	17.000	11.000	17.000

Subsequently, each pre-test question item was analysed. The proportion of accurate, partially correct, and incorrect responses to each question is depicted in Figure 4. This shows that most students correctly solved the first four problems that did not require rearranging terms containing variables. Subsequent questions generated increasingly incorrect responses, particularly questions 7 and 8, where only three students submitted correct responses. A similar tendency was observed for questions 5 and 6 when at least 23 students provided incorrect responses. At this point in the learning process, it is expected to anticipate that students will have a more significant propensity to struggle when solving algebraic equations that require rearranging the terms before finding the solution.

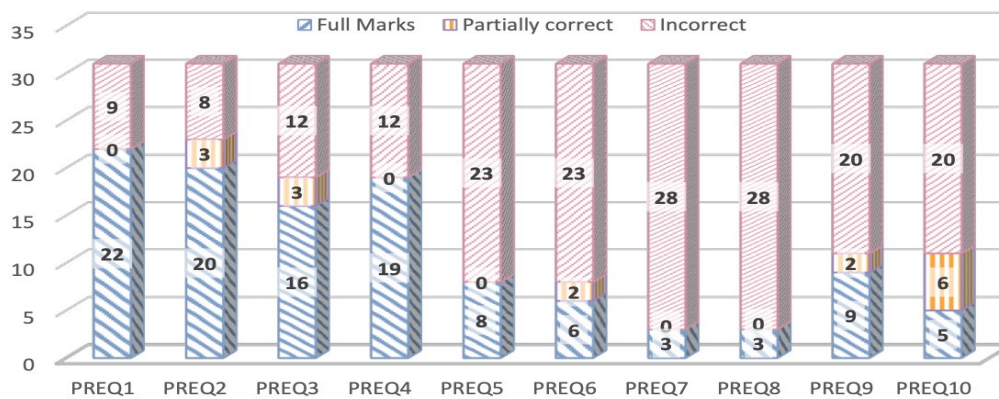


Figure 4. The Item Analysis of Pre-test Scores for Overall Samples.

Descriptive Statistics of Post-Test Results

Table 4 presents the descriptive statistics for the post-test results of the entire sample, class Aster and class Buttercups. The total mean post-test scores for both classes are above 11, with class Aster's post-test scores ranging from 6 to 19 and class Buttercup's post-test scores ranging from 2 to 18. Hence, the range of total post-test scores for both groups is from 2 to 19. Class Buttercups has the most significant standard deviation at 5.557, followed by the entire sample at 4.982 and class Aster at 4.457.

Table 4

Post-test Descriptive Statistics for Class Aster, Class Buttercups, and Overall Sample

	Class Aster	Class Buttercups	Overall Sample
Valid	16	15	31
Mean	12.563	11.200	11.903
Std. Deviation	4.457	5.557	4.982
P-value of Shapiro-Wilk	0.153	0.133	0.049
Minimum	6.000	2.000	2.000
Maximum	19.000	18.000	19.000

The post-test questions were then evaluated by tallying the number of students who answered each question correctly, partially, or inaccurately. From Figure 5, 16 out of 31

students scored at least one mark on each post-test question. In addition, items 1, 5, 6, and 7 have the most significant number of appropriate responses, with 28, 24, 28, and 25 students, respectively, getting perfect scores. There were also some incorrect responses (zero points) for each question, with a maximum of 15 students for items 2 and 3. Item 6 has the fewest incorrect responses, with only one student failing to obtain any credit for this question that requires more than two steps to solve linear equations.

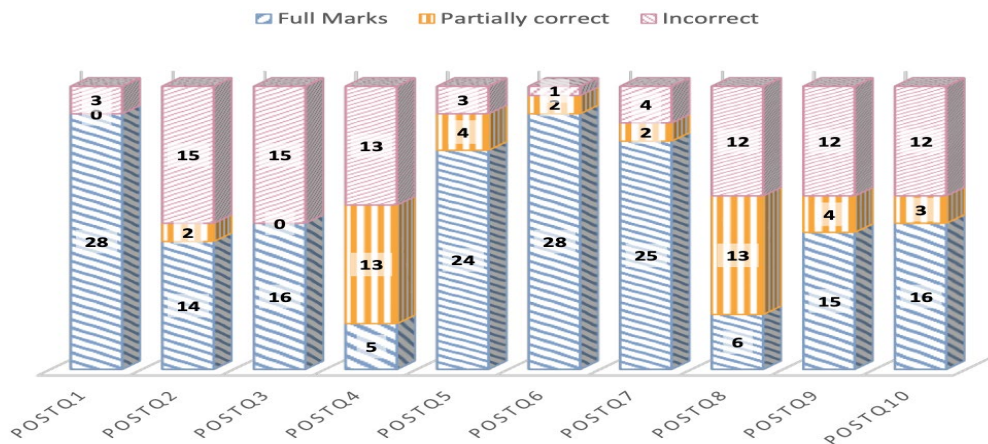


Figure 5. The Item Analysis of Post-Test Scores for Overall Samples.

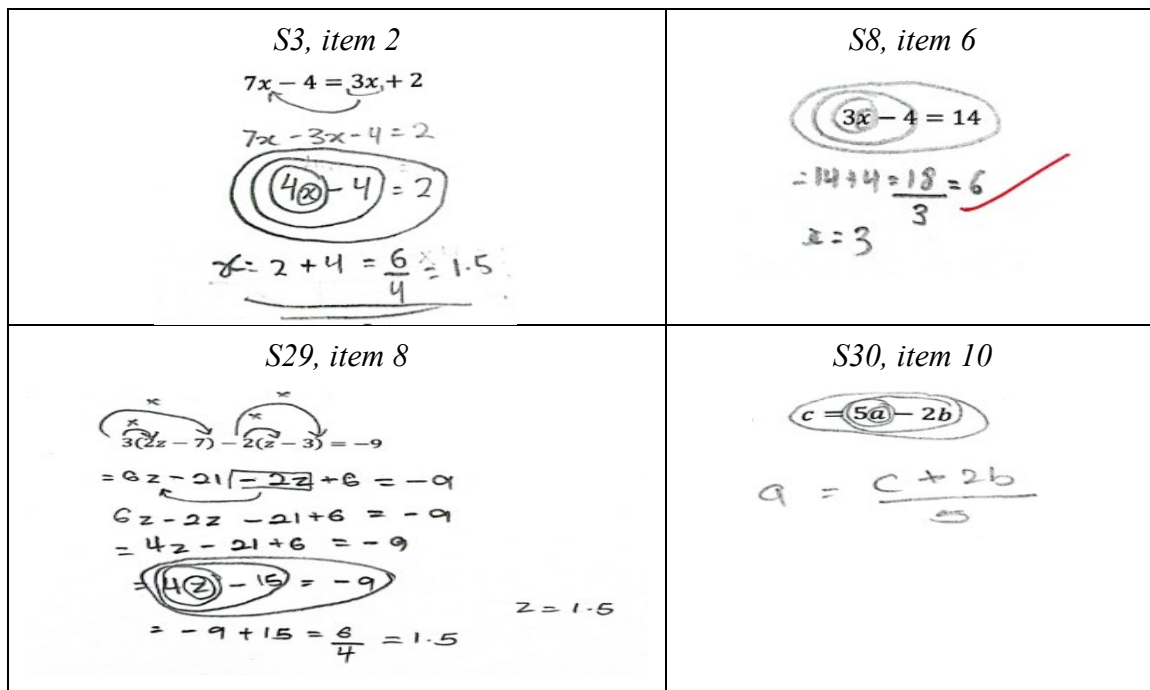


Figure 6. Samples of Students' Post-Test Responses Using the OSM.

During the post-test evaluation, students from classes Aster and Buttercups attempted the items using the new OSM method. The students S3, S8, S29, and S30 attempted items 2, 6, and 10, respectively (Figure 6). After simplifying the equations, the students effectively implemented the OSM and demonstrated their efforts by sketching onion skins to solve basic linear equations.

Descriptive Statistics of Pre-Test and Post-Test Results

According to Table 5, the average percentage point between pre-test and post-test results for all samples increased to 11.903 from 7.000, which indicates the mean values have moved from an average range of marks to a higher range of marks. The mark difference has the lowest standard deviation as measured by the standard deviation, indicating that the marks are dispersed around the mean.

Table 5
The Pre-and Post-Tests and the Mark Difference Descriptive Statistics for the Overall Sample

	Pre-test Total	Post-test Total	Marks Difference Total
Valid	31	31	31
Mean	7.000	11.903	4.903
Std. Deviation	4.837	4.982	4.714
Shapiro-Wilk	0.952	0.932	0.965
P-value of Shapiro-Wilk	0.172	0.049	0.393
Minimum	0.000	2.000	-5.000
Maximum	17.000	19.000	13.000

The Significant Difference in the Pre-test and Post-test Scores

If the underlying assumption was valid, the pre-test and post-test results were evaluated using a paired sample t-test; otherwise, a Wilcoxon Signed Rank test was used to assess a significant difference between pre-test and post-test performance. The preceding methodology section discussed the first, second, and third assumptions. The Shapiro-Wilk test is utilised to evaluate the fourth assumption of normal distribution of the difference between the pre-test and post-test. The Shapiro-Wilk test in Table 5 demonstrates that the p-value for the difference between the pre-test and post-test is more than 0.05, indicating that the data are typically distributed. Thus, the null hypothesis of having no substantial difference in student performance in both tests is rejected. The boxplot in Figure 7 demonstrates no outlier in the difference between pre-test and post-test findings; hence, the fifth assumption is valid. A paired sample t-test was performed to compare the median scores on the pre-test and post-test, given that all of the hypotheses were satisfied.

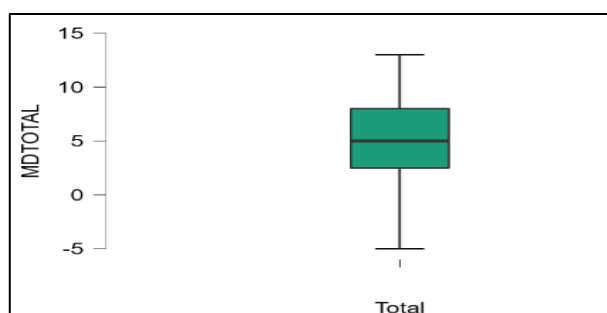


Figure 7. The Boxplot of the Mark Difference in the Pre-Test and Post-Test Scores.

A paired samples t-test was performed to assess the significance of the score difference between the pre- and post-tests. The results revealed a statistical increase in between the scores from the pre-test ($M=7.000$, $SD= 4.837$) to the post-test ($M=11.903$, $SD= 4.982$), $t(30)=5.791$, $p <.0005$ (two-tailed). The mean score increase was 4.903, with a 95% confidence interval ranging from 6.632 to 3.174. Furthermore, the effect size of Cohen’s d is more than 0.8, which indicates a large effect size (Cohen, 2013). This suggests that OSM impacted students’ ability to solve linear equations.

The pre-test and post-test scores were compared, as illustrated by the graph (see Figure 8), and the distribution has become more negatively skewed on the post-test. Most students’ grades have shifted to the relatively high 12-to-19-point range. This discovery adds to the evidence that the mean score increased, indicating that students performed better on the pre-test than on the post-test. In addition, the number of students who scored 0 to 1 mark decreased significantly, while those who earned at least 13 increased significantly. As a result, the bar graph depicts an upward trend in the post-test performance distribution of students.

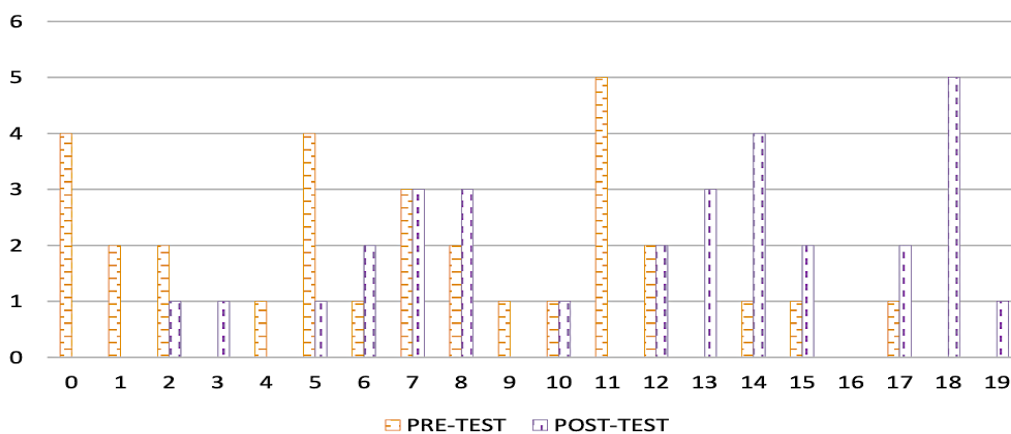


Figure 8. The Distribution of Pre-Test and Post-Test Total Scores for Overall Samples.

Each student’s performance on the pre-test and post-test was compared, with special consideration given to those who received faultless scores on every item. Since the only difference between the pre-test and post-test items is their order (i.e., pre-test item 2 = post-test item 6), the identical items are compared (Figure 9). The line graph demonstrates that the proportion of correct answers to each question rose over time. Item 6 of the pre-test, which needed three steps to solve an equation, and item 10 of the pre-test, which required the rearrangement of an algebraic problem, had the most significant improvement, with ten more students obtaining higher scores on the post-test than on the pre-test. In addition, as indicated earlier, nearly all students utilised the OSM on their post-test, indicating that this approach enhanced students’ confidence to make a correct attempt on each question.

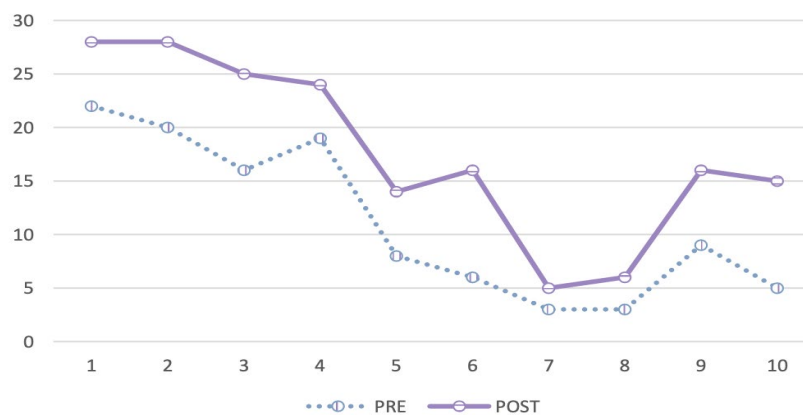


Figure 9. Students Achieving Perfect Scores in Pre- and Post-Tests.

In response to the first research question, how does the OSM facilitate students' solving linear equations? The result showed that the OSM positively impacts students' performance in solving linear equations. The mean pre-test scores for the overall marked at 7.000, class Aster at 9.875, and class Buttercups at 3.933, while the mean post-test scores for the overall, class Aster, and class Buttercups are 11.903, 12.563, and 11.200, respectively. This demonstrates that the mean values for all categories have increased significantly.

The line graphs (Figure 9) demonstrated a clear depiction of student performance improvement, with the graphs shifting towards higher marks. After the intervention using the OSM, the number of students earning nine marks (the passing line) and below decreased considerably, while the number of students achieving 10 to 19 increased significantly. The bar graph likewise shows improvements in the outcomes for each item's mark. After the intervention, the pre-test and post-test mean scores were compared using a paired samples t-test to determine if there was a significant difference between students' performance. The null hypothesis that states no significant difference between students' performance was rejected as the findings demonstrated a statistical difference in students' performance in the pre-test and post-test scores for the entire sample with a p-value of more than 0.05 (0.393). Moreover, the effect size exhibited a large effect size. As a result, OSM significantly improves students' performance in solving simple linear equations.

Semi-Structured Interviews

Table 6 provides details of the semi-structured interview with six selected students. In this study, the interview questions were used as the main basis for categorising and identifying themes. The responses provided extensive data for subsequent careful analysis. The interview questions were designed to determine specific aspects of the topic. By organising the responses according to the questions, a focused yet comprehensive approach to data analysis was adopted. The interview interactions were transcribed and analysed further, and then each code was categorised according to emerging themes from the data in response to the specific questions posed to the students. The interview generated five distinct themes. Some interviews were conducted in both English and Malay. However, for convenience of discussion, the excerpt contains only translated quotations.

Table 6

Details of Students Chosen for the Semi-structured Interviews

Performance	Students
1. Students that have higher overall performance on the pre-test	S10 & S16
2. Students that have a better overall performance on the post-test	S8 & S29
3. Students who maintained their performance level from the pre-test to the post-test	S2 & S28

Theme 1: Time spent by students on solving simple linear equations

The six respondents had a range of opinions regarding the time it took them to finish the pre-test and post-test papers (Figure 10). S2, S8, S10, and S16 reported that they completed the post-test evaluations more quickly; however, S28 indicated that she took the same amount of time to complete both papers. S29’s response was completely different from the others; she claimed that she completed her pre-test assessment faster than her post-test assessment and that the method employed in each evaluation contributed to this difference.

Teacher: How fast do you think you can solve simple linear equations during the ...	
Pre-Test?	Post-Test?
S2: 10 to 15 minutes?	S2: It’s the same. This one <post-test paper> (is faster).
S8: Erm... below 30 minutes.	S8: I think the post-test is faster. I think less than 15 minutes.
S10: Erm... 20 minutes.	S10: Around 10 to 15 minutes.
S16: 15 to 20 minutes?	S16: The same. But I think post-test is faster.
S28: Erm... Maybe around one hour?	S28: I think the same.
S29: 10 minutes.	S29: 15 minutes.

Figure 10. Interview Excerpts for Theme 1.

As mentioned earlier, students were likely exposed to the *transposing method* when solving algebraic equations (Damit et al., 2017). Therefore, it must be accepted that all students have substantial experience employing the method in question. In contrast, the new method was introduced within the same week as the post-test, when they were still familiarising themselves with its procedure steps. Despite this, most students continue to believe that the OSM has greatly assisted them in enhancing their scores. The students embraced the teacher’s conceptions of knowledge, reproduced a new strategy, and replicated it for the test and evaluation (Yunus et al., 2016).

Theme 2: Students’ reactions to their performance on the post-test

The respondents were also questioned regarding their assessment performance (Figure 11). S8 believed that the new strategy enhanced her performance on the post-test. S29 acknowledged she needed to prepare for the pre-test despite having studied the topic before the assessment. At the end of the statement, she also said that she would always be ready if requested to retake her post-test, as she had now memorised the steps of the new method. In a sense, OSM can be used to encourage students to ruminate on their knowledge and make better use of available concepts, as well as their existing knowledge of transposing the correct term when solving simple linear equations (Sarwadi & Shahrill, 2014). When S2 and S28 were interviewed to compare their pre and post-test performance, S2 claimed that her identical method selection on

both tests contributed to her scores, and she stated that she might have improved her post-test score by using the OSM. Yet, S28 was uncertain as to why she had a similar performance on the post-test.

Teacher: Why do you think you perform better in the ...	
Post-Test?	Pre-Test?
S8: Because I used the onion skin method.	S10: Erm... The re-arrangement of the terms as well as forgot to change to the opposite signs.
S29: For this <pre-test> we had a surprised test. As for this <post-test>, we still have enough time to do revision once we learn the steps.	S16: [Flipping page]. I think my post-test will be a little bit better if I calculate correctly.
Teacher: Why do you think you perform equally in both tests?	
S2: Because I used the same method for both tests.	
S28: I am not sure.	

Figure 11. Interview Excerpts for Theme 2.

Theme 3: Students' preferred method in solving linear equations

Almost all the students preferred the OSM over the method previously used in the pre-test (Figure 12). For example, despite seeing a decline in her scores, S16 preferred employing the OSM. Yet, S28 claimed to be adaptable and had no difficulty executing both procedures. However, teachers must understand students' learning patterns, which are relatively stable indicators of how students interpret the lesson and their capacity to process knowledge successfully (Belecina & Ocampo, 2019; Coetsee & Kakoma, 2017; Egodawatte, 2009; Pallapu, 2007; Reid, 1987; Te Maro et al., 2008).

Teacher: Which method do you prefer?
S2: Onion skin.
S8: Onion Skin.
S10: Onion skin method.
S16: The onion method.
S28: Both.
S29: Onion skin.

Figure 12. Interview Excerpts for Theme 3.

Theme 4: Students' perception of the OSM

When questioned about their view of the OSM, every respondent responded favourably (Figure 13). S8 and S10 commented on their claims that the OSM helped them determine which term should be transposed first. While S2, S28, and S29 concur that the strategy is simple to follow and has memorised the steps.

Teacher: What do you think about the onion skin method?
S2: Because it is easy to solve the equations. It is easier for me. The steps are easier for me to remember.
S8: Erm... Because I know what I need to do according to the sequence.
S10: It is separated so it is easier to know which one comes first
S28: Uhh... Easy. Because I managed to do it to solve question.
S29: Because like onion skin skill method, it's easy to solve it.

Figure 13. Interview Excerpts for Theme 4.

After asking S16 about her preferred method, another distinct question was posed (Figure 14). She noted that her previous method (*transposition method*) left her needing clarification about the transposition sequence. The OSM employed a mnemonic tactic in solving linear equations, providing a more significant benefit, especially for visual learners (Coetsee & Kakoma, 2017; Emran et al., 2023; Pallapu, 2007). It is anticipated that this OSM has helped students better understand the significance of transposing the correct order when solving simple linear equations, which is one of the primary reasons most students picked the OSM over the *transposition method*.

As Sarwadi and Shahrill (2014) stated, many mathematical concepts cannot be comprehended until the learner has mastered the concept's application procedures. Similar to peeling back the layers of an onion, the OSM gradually introduces algebra. Students commence with fundamental concepts and progressively advance to more complex subjects, such as factoring and solving equations. This gradual approach helps them build a solid understanding, making it easier to grasp more challenging material as they advance (Widener, 2014).

The OSM supports steady skill development by breaking complex ideas into smaller, more manageable steps. Thinking of algebra as layers helps students recognise how different concepts are interconnected, reinforcing their comprehension. On the other hand, traditional teaching methods, such as direct instruction, inquiry-based learning, and problem-solving, often focus on presenting concepts through explanations and examples. While these approaches provide clear direction, they may not always help students see the relationships between different algebraic ideas or fully grasp the subject's broader structure. As a result, students might learn how to solve specific problems but struggle with more profound understanding and long-term retention, especially if they are not actively engaged in the learning process.

Teacher: Why do you think you prefer the onion skin method than the transposing method?
S16: Because it is harder?
T: Which one?
S16: The pre-test.
T: You think the transposing is harder?
S16: [Nodding]
T: In what term?
S16: Sometimes it makes me a little bit confusing which one to move first.

Figure 14. Interview Excerpts with Student S16.

Theme 5: Students' future use of the OSM

The six students interviewed were asked whether they would consider employing OSM for future learning (Figure 15). S28 claimed that she could solve simple linear equations using any

approach. Fundamentally, all respondents responded positively, indicating they would use the method in the future. S16 indicated that she could perform better if she had more opportunities to use OSM. In addition, S10 has demonstrated her confidence in achieving a perfect score by incorporating the OSM if she were given a second chance to retake her post-test (Figure 16).

<i>Teacher: Would you use the onion skin method in the future?</i>
S2: Yes.
S8: Yes.
S10: Yes.
S16: Maybe I will. If enough practice, maybe I can do better.
S28: Both

Figure 15. Interview Excerpts for Theme 5.

<i>Teacher: Would you use the onion skin method in the future?</i>
S10: Yes.
T: Even when I'm not around?
S10: Yes.
T: So, if you're given the opportunity to re-do the post-test, are you confident in getting better marks?
S10: Yes. Full marks maybe.

Figure 16. Interview Excerpts with Student S10.

In response to the second research question, how do students perceive integrating the OSM in solving a simple linear equation? Five themes were generated from the interviews. The first theme pertains to the length of time to complete their assessments. Four out of six students said they completed their post-test in less time than their pre-test, while the other two claimed that they either took the same amount of time or that their pre-test time was significantly faster. The interviewed students who enhanced their scores from the pre-test to the post-test claimed that using OSM during the post-test contributed to their score improvement. Similarly, one of the students who kept their pre and post-test scores admitted to utilising the identical transposition approach for both tests. She indicated that there would be a change in her post-test scores if she utilised the OSM procedure.

All the interviewed students agreed that the OSM is simple since the procedures for choosing which term must be transposed first are more transparent once the onion skin is created and, thus, more accessible to memorise. In addition, all the students interviewed were confident in their ability to use OSM, and five out of six students favoured the OSM over their previously taught method. Only one of the students claimed to be adaptable to any method of solving linear equations. Interestingly, all six students said they would employ the OSM for future use. According to previous studies, many children are visual learners (Coetsee & Kakoma, 2017; Pallapu, 2007). The OSM has the potential to assist students who learn best through visuals in translating algebraic equations into visual representations, thus improving their time in responding to the questions using the *transposing method*.

Due to its flexibility, OSM has the potential to accommodate a diverse range of students. For students who struggle with algebra, the layered approach provides structured guidance,

reducing cognitive overload by breaking problems into smaller, more digestible steps (Abay & Clores, 2022; Booth et al., 2016; Nofrianto et al., 2022; Pallapu, 2007). This is particularly useful for learners with learning difficulties like dyscalculia, who benefit from a straightforward, step-by-step breakdown of mathematical operations.

Enrichment activities such as incorporating advanced algebra or real-world problem-solving can be integrated into OSM for advanced students. This enables gifted students to acquire knowledge more extensively while adhering to a systematic methodology. The OSM provides multiple entry points into a subject through differentiated instruction. Visual learners may prefer layered diagrams to represent algebraic processes, while verbal learners may choose written explanations. Interactive tools, including digital simulations and manipulatives, effectively engage kinesthetic learners (Pallapu, 2007; Salihi et al., 2020).

Conclusion

Most students in this study perceived that the OSM enables them to solve linear equations not only correctly and precisely but also quickly and efficiently, mainly because they know the correct sequence of the term that needs to be transposed. Furthermore, the students in the study prefer to utilise the OSM over their previous method since it is easy to comprehend, visualise, and remember the processes. The paired sample t-test results indicated that the entire group's median score increased significantly between the pre-test and post-test. In addition, the item-by-item analysis demonstrated an increase in correct responses per item with at least one mark and a reduction in incorrect responses.

The study discovered that the OSM improved students' ability to solve basic linear equations. The improvement was particularly noticeable in one of the three problem-solving phases and the rearrangement of an algebraic equation. Additionally, the semi-structured interviews revealed that most students have a favourable view or attitude towards the intervention strategy. The mnemonic pattern of the onion skin enabled students to visualise the term of the equation that needed to be transposed to the other side, reducing the likelihood of errors. Furthermore, due to its simplicity, students will have no issue learning and remembering the procedures required to use this strategy.

Nevertheless, the OSM could confuse some students, especially when inverting the operation of "peeling" the onion skin. More practice could rectify this situation. Despite this, most students decided that OSM would be most beneficial for future applications in mastering the topic of linear equations.

Educators can improve the effectiveness and inclusivity of algebra instruction by incorporating the OSM into comprehensive educational frameworks and tailoring them to cater to the needs of diverse students. The sequential format helps struggling students understand the material better, while its adaptability allows advanced students to investigate concepts more deeply. Educators can also incorporate the OSM into the curriculum to improve students' understanding and retention of algebraic concepts. Consequently, it is proposed that OSM has the potential to serve as a valuable tool for developing structured and effective mathematical reasoning.

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