Southeast Asian Mathematics Education Journal
2016, Vol. 6 No. 1, 59-68

Impact of Brain Research upon School Mathematics for the 21st Century

Allan Leslie White
University of Western Sydney, Australia.

Abstract

In 1924, Hans Berger succeeded in recording the first human electroencephalogram (EEG). With developments in technology, there are now a variety of approaches for examining brain activity such as Magnetic resonance imaging (MRI), nuclear magnetic resonance imaging (NMRI), magnetic resonance tomography (MRT) and computed tomography (CT scans). This technology is giving the first glimpses of the vastness of our inner brain space and brain research is being used to treat autism spectrum disorders, Alzheimer's disease, Parkinson's disease and other brain related conditions. Importantly, the implications of brain research for education are beginning to emerge. This paper will discuss some of these implications with special focus upon mathematics teaching and learning. It will discuss a scale for classifying teaching strategies according to their aims and student learning outcomes involving understanding. It will end with a reflection on the importance of the end of a mathematics classroom lesson.

Keywords: school mathematics, insight, instrumental understanding, rote memorization, relational understanding.

Introduction

In order for teachers to communicate a mathematical idea, a representation is needed. This representation can involve spoken language, written symbols (numbers, algebra, etc.), pictures (photos, graphs, etc.), video, dynamic images or physical objects. For students to receive this representation they must think, and from a cognitive science viewpoint, this thinking produces an internal representation. Thus a problem arises for all mathematics teachers in assessing the quality of this internal representation of the student.

This problem has been tackled by many researchers across many disciplines such as philosophy and psychology. The early Behavourists (see Skinner, 1953) rejected the idea of internal representation because it could not be observed. For other groups, such as Constructivists, there were various strategies used for making inferences about the quality of these internal representations (or constructions) yet none of these strategies have actually involved observing what happens in the brains of the students when they are thinking. Add to this the fact that the students are often unable to articulate their thinking.

> We all have systems of concepts that we use in thinking, but we cannot consciously inspect our conceptual inventory. We all draw conclusions instantly in conversation, but we cannot consciously look at each inference and our own inference-drawing mechanisms while we are in the act of inferring on a massive scale second by second. We all speak in a language that has a grammar, but we do not consciously put sentences together word by word, checking consciously that we are following the grammatical rules of our language. To us, it seems

> easy: We just talk, and listen, and draw inferences without effort. But what goes on in our minds behind the scenes is enormously complex and largely unavailable to us (Lakoff & Nunez, 2000, p.27).

It is the newly emerging field of brain research that offers new insights into the actual workings of the brain. In 1924, Hans Berger succeeded in recording the first human electroencephalogram (EEG). Since then, there have been significant developments in technology, and now there are a variety of approaches for examining brain activity such as Magnetic resonance imaging (MRI), nuclear magnetic resonance imaging (NMRI), magnetic resonance tomography (MRT) and computed tomography (CT scans). These and other technologies are giving researchers the first glimpses of the vastness of our inner space just as super telescopes are mapping outer space. Brain research has impacted upon medicine and is being used to treat autism spectrum disorders, Alzheimer's disease, Parkinson's disease and other brain related conditions. Importantly for this paper, the implications of brain research for mathematics education are beginning to emerge. Brain research provides a fresh perspective on the teaching and learning of mathematics. In the following sections I will attempt to briefly present a small sample of the findings that have relevance for mathematics education in the twenty-first century with a special focus upon school mathematics teaching and learning

Learning and Understanding

From a theoretical viewpoint, cognitive science makes two assumptions in the communication process between the teacher's external representation and the student's internal construction of that representation. First they assume there is a relationship between the external and internal representations (the student's construction), and secondly the internal representation can be connected to other representations in useful ways (connected knowledge or schemas).

From a brain research viewpoint, the process of learning begins when neurons form networks that fire together. The more an individual uses the networks the more developed they become until eventually they become automatic. Conversely, through less use the networks decay and eventually become lost. This key concept has been termed 'brain plasticity' or 'neuroplasticity', which refers to the ability of the brain to change. Research has shown that the brain can reorganise itself in remarkable ways as a result of a change in stimuli. It is essentially a process of rewiring the brain by forming or strengthening new connections and allowing old connections to decay. Brain researchers have shown that:

> Children are not always stuck with mental abilities they are born with; that the damaged brain can often reorganise itself so that when one part fails, another can often substitute; … One of these scientists even showed that thinking, learning, and acting can turn our genes on and off, thus shaping our brain anatomy and our behaviour (Doidge, 2008, p. xv).

The implications for education, and especially school mathematics are profound. It opposes the traditional beliefs that some children are born with the ability to do mathematics, others are not.

> … scientists now know that any brain differences present at birth are eclipsed by the learning experiences we have from birth onward (Boaler, 2016, p. 5).

Children are not born knowing mathematics. They are born with the potential to learn mathematics. How this potential is nurtured, encouraged, and challenged is the responsibility of parents and teachers.

> Students can grasp high-level ideas, but they will not develop the brain connections that allow them to do so if they are given low-level work and negative messages about their own potential. (Boaler, 2015, p. xvii)

When a child learns a new concept, an electric signal sparks, cutting across synapses and connecting different parts of the brain. If the child learns a concept deeply, then the synaptic activity creates lasting connections in the child's brain, whereas surface learning quickly decays. How this decay occurs was outlined by Sousa (2008) who stated that scientists currently believe there are two types of temporary memory. Firstly, immediate memory is the place where the brain stores information briefly until the learner decides what to do with it. Information remains here for about 30 seconds after which it is lost from the memory as unimportant. Secondly, the working memory is the place where the brain stores information for a limited time of 10 to 20 minutes usually but sometimes longer as it is being processed. The transfer from immediate memory to working memory occurs when the learner makes a judgement that the information makes sense or is relevant. If the information either makes sense or is relevant then it is likely to be transferred to the working memory, and if it has both then it is almost certain to be transferred to the long-term memory.

So one area of importance to how students make this judgement of relevance is the area of attitudes and beliefs around learning and how the brain worked. The school mathematics curriculum includes many facts, skills, procedures and concepts. Mathematics teachers are expected to teach the curriculum while inculcating positive attitudes towards mathematics and by engaging and motivating their students to work mathematically. Psychologist Barbara Dweck (2006) and her research team collected data over a number of years and concluded that everyone held a core belief about their learning and their brain. They made a distinction between what they labelled as a fixed mindset and a growth mindset. Someone with a fixed mindset believes that while they can learn things, they cannot change their intelligence level. Whereas someone with a growth mindset believes that the brain can be changed through hard work and the more a person struggles the smarter they become. There is an obvious connection here between growth mindset and brain plasticity. Professor Jo Boaler (2016) in her new book provides a wealth of research evidence involving mathematics learning that supports Dweck's work.

> It turns out that even believing you are smart - one of the fixed mindset messages - is damaging, as students with this fixed mindset are less willing to try more challenging work or subjects because they are afraid of slipping up and no longer being seen as smart. Students with a growth mindset take on hard work, and they view mistakes as a challenge and motivation to do more (Boaler, 2016, p. 7)

Boaler and her team have developed a website (Youcubed), produced many short videos (search for Jo Boaler on Youtube for a selection), and published considerable material on how to promote growth mindsets in the classroom. They list seven positive norms for teachers to promote in their classrooms (Boaler, 2015; 2016, pp. 269-277). They are: Everyone can learn mathematics to the highest level; Mistakes are valuable; Questions are really important; Mathematics is about creativity and making sense; Mathematics is about connections and communicating; mathematics is about learning and performing; and the final norm is that depth is more important than speed.

My aim for the rest of this paper is not to replicate or summarise Boaler's material, as the reader can get access to it through the links I have mentioned. Instead I want to continue considering the implications of the Scale for Teaching for Understanding and current brain research for school mathematics teaching and learning. This scale attempts to classify teaching strategies according to the lesson aims and student learning outcomes regarding success at constructing meaning. In some areas, the paper will overlap or resonate with some of Boaler's classroom norms. So in the following section I will briefly provide an overview of the scale for teaching for understanding before elaborating upon mathematical insight.

Scale for Teaching for Understanding

A classroom teacher is faced with a vast array of teaching strategies from which to choose. In my earlier papers (White, 2011, 2013) the negative effects of behaviourism, rote memorisation and skills based teaching strategies for teaching mathematics have been discussed in some detail. These are a few of the many strategies regarded as ineffective or even harmful to the development of mathematical understanding. Why is understanding or constructing meaning so important? Meaning determines the possibility that information will be learned and retained in the long term memory, the goal of all mathematics teaching and learning. As mentioned earlier, making sense or meaning is a crucial consideration of the learner in moving information to both the working and long term memory.

> Students may diligently follow the teacher's instructions to memorize facts or perform a sequence of tasks repeatedly, and may even get the correct answers. But if they have not found meaning by the end of the learning episode, there is little likelihood of long-term storage (Sousa, 2008, p. 56).

It appears that making sense, meaning or understanding does not have a single end point but refers to a process of an increasing accumulation of input and connection. Research literature contains a number of duopolies such as Skemp (1976, 1977, 1979, 1986, 1989, 1992) who proposed the terms instrumental and relational understanding. Instrumental mathematical understanding is described as 'rules without reasons' or knowing how or what to do to get an answer, whereas relational understanding is concerned with meaning and knowing both what to do and why it is being done to get an answer. Skemp (1976, 1977) discusses the developing of schemas as evidence of the construction of relational understanding and this resonates very strongly with the structure of the connections within the brain and with the research literature on 'connected knowledge'.

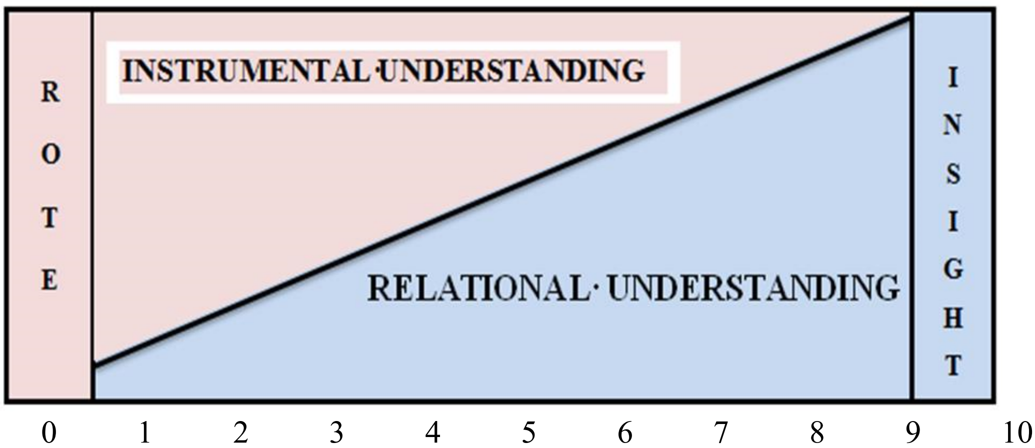

Figure 1. Teaching For Understanding

The scale of teaching for understanding was constructed as a continuum (see Figure 1) based on the assumptions that all teaching strategies can be classified according to their aims and outcomes using Skemp's types of understanding, and that the struggle to assist learners to understand is the struggle to make sense or meaning. This scale has been discussed more fully elsewhere (White, 2014).

Briefly, the left end of the scale (score 0) is the most extreme end of instrumental teaching strategies which is rote memorization, where there is no attempt to assist students to understand or connect what they are memorizing with what they already know.

Sousa (2008) contrasts two kinds of practice as rote and elaborative rehearsal regarding their effects on the brain. Rote rehearsal is a process of learning information in a fixed way without meaning and is easily forgotten. Elaborative rehearsal encourages learners to form links between new and prior learning, to detect patterns and relationships and construct meaning. The construction of meaning involves the building of cognitive schemas that will assist long term memory. Elaborative rehearsal leads to meaningful, long-term learning. Of course there are a range of elaborative rehearsal teaching strategies that differ in success.

As the scale indicates with scores of 1 to 9, for the majority of teaching strategies, teaching for understanding involves a combination of instrumental, relational and memory strategies and elaborative rehearsal that are all important in the process of building more sophisticated concepts that are meaningful to the learner. When we consider the time allocated to practice or rehearse then there is another distinction made in the literature between *massed practice* and *distributed practice* (Sousa, 2008). Cramming, which usually occurs in a brief intense time period just before an examination, is an example of massed practice where material is crammed into the working memory, but is quickly forgotten without further sustained practice. There is no sense making and so it never makes it into the long term memory. Distributed practice on the other hand is sustained practice over time, building understanding and resulting in long-term storage. Distributive practice resonates very strongly with the East Asian Repetitive Learning which is continuous practice with increasing variation as a route to understanding (Leung, 2014), and this is often misunderstood as a form of rote. It is not, as it seeks to build understanding through increasing the complexity and the connections with prior knowledge.

The interplay of the instrumental and relational aspects of understanding leads to what is often termed compression which is also sometimes confused with rote.

> Mathematics is amazingly compressible:-you may struggle a long time, step by step, to work through the same process or idea from several approaches. But once you really understand it and have the mental perspective to see it as a whole, there is often a tremendous mental compression. You can file it away, recall it quickly and completely when you need it, and use it as just one step in some other mental process. The insight that goes with this compression is one of the real joys of mathematics (Thurston, 1990, p. 847).

As Thurston (1990) states above, the process of building understanding leads to insight. Insight was apparently derived from a Dutch word for 'seeing inside' and is loosely defined as the process within the mind of a learner who when exposed to new information enables the learner to grasp the core or essential features of a known problem or phenomena. An insight seems to result in a connective process within the brain or a quick restructuring that produces new understanding that is a compression of the connected information. Thus encouraging student insight is a goal in the process of teaching for understanding.

Researchers draw strong connections between insight, creativity and exceptional abilities, with any significant and exceptional intellectual accomplishment almost always involving intellectual insights (Sternberg, 1985). Mathematical insight has long been recognised as a feature of many mathematical greats. Ramanujan, the 1930's famous mathematician from India who was essentially self-taught amazed his European colleagues with his flashes of insight. His time spent at Cambridge was a struggle to prove theorems that:

> … came to him so readily, whether through the divine offices of the goddess Namagiri, as he insisted, or through what Westerners might ascribe, with equal imprecision, to intuition' (Kanigel, 1991, p. 215).

This is not the time to discuss the distraction of whether mathematical reality exists independently in the world (and imparted by gods or goddesses), a language of the universe awaiting discovery by mathematicians or on the other hand, if it exists as a construction of the collective brains of mathematicians as was more likely the favoured view of the English pure mathematician G. H Hardy who collaborated with Ramanujan on partitions. Instead, we maintain focus on insight for Hardy marvelled at Ramanujan's ability to produce theorems that connected mathematics which had been widely considered independent and unrelated:

> … there seems no escape, at least, from the conclusion that the discovery of the correct form was a single stroke of insight. (Kanigel, 1991, p. 281).

Insights can occur as a result of the conscious and unconscious mind. The unconscious mind can continue to operate when the conscious mind is otherwise distracted, hence the large number of cases of mathematics students claiming to have gone to bed with an unsolved mathematics problem only to wake the next morning with an insight into the solution.

> Perhaps the most fundamental, and initially the most startling, result in cognitive science is that most of our thought is unconscious that is, fundamentally inaccessible to our direct, conscious introspection. Most everyday thinking occurs too fast and at too low a level in the mind to be thus accessible. Most cognition happens backstage. That includes mathematical cognition (Lakoff & Nunez, 2000, p.27).

An insight is not an end in itself but can contribute to further understanding and further insight.

> A student in year 7 may come to an understanding of Pythagoras' theorem where the area of semi-circles drawn on the triangle with sides as diameters is seen as also obeying the theorem. Later in year 10, the same student may develop a further insight where Pythagoras' theorem is seen as one example (one angle equal to 90 degrees) of the more general rule known as the cosine rule (for any angles). Thus mathematics teaching that leads to the production of multiple insights in the learner is postulated as a desirable goal for the teacher. (White, 2014, pp. 65-6).

It is the accumulation of insights that leads to the desired compression of mathematical understanding. This compression provides the mathematical tools to efficiently tackle more sophisticated and complicated mathematical problems.

In the above sections a brief discussion of the importance of sense or meaning making has been presented in the light of the emerging brain research. I would like to conclude this brief paper with a discussion of the implications of this discussion above upon a part of a mathematics lesson that is often ignored or left unplanned, and that is the lesson closure. The end of the mathematics lesson.

Closure: A Forgotten Step in Helping Students Construct Meaning

We have seen earlier in this paper the crucial student learning bond between retention and construction of meaning. Brain research also supports the assertion that in any 40 minute lesson, there are optimal times for student learning and for constructing meaning. Students tend to remember best what comes first and second best what comes last and this is known as the *primacy-recency effect.* Sousa (2008) provided the diagram below for a forty minute lesson.

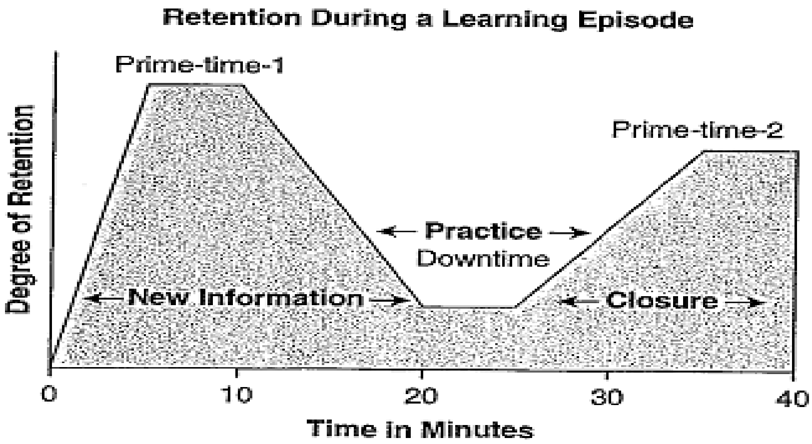

Figure 2. Times For Optimal Retention. (Sousa, 2008, p. 61).

In the first period of prime time of about ten minutes, there is firstly an increase of retention that would correspond to the teacher introducing the new material in an interesting and challenging way. The prime time is the meaning constructing phase where the students are assisted in making sense of the new material and connecting it with their existing knowledge. Here retention is at as maximum.

In the middle of the lesson there is a less crucial period where students would practice applying this new material. As stated earlier, if the drill and practice material is well structured and relevant, then it can also contribute to deepening the student's understanding. So retention is still happening and so the graph does not return to the axis. It is just not as great as before.

Finally, the end of the lesson involves closure. When I was a young teacher many mathematics lessons ended with drill and practice while I helped students independently. While this may help deepen understanding for some students it is not very efficient. My inexperience often led to the lesson suddenly ending with the bell. I was unaware then of the importance of closure, although with experience I was able to better manage classroom time and plan a conclusion. The question arises: What can teachers do to make optimal use of the five minutes at the end of the lesson in order to help students connect their thinking and improve retention?

As closure focuses upon further construction of meaning through connections with prior learning, use can be made of strategies involving the building up of network summaries on the board from class contributions. Strategies such as mind maps, flowcharts, concept maps, are just a few of many that are available. The use of strategies involving student writing are another group. Having students write for five minutes at the end of every mathematics lesson can serve two purposes of assisting the student to clarify their thinking while also providing to teachers, another source of assessment for learning classroom data.

Seeing that this section has dealt with the conclusion of a lesson, it seems appropriate to move to the conclusion of this paper.

Conclusion

This paper has sought to discuss some of the findings that brain research is providing to the teaching and learning of mathematics. It seeks to motivate mathematics teachers to develop strategies that encourage students to build their mathematical understanding and develop links and connections within their knowledge, while developing their skills and positive attitudes towards their mathematical learning and knowledge. The paper also briefly highlights the complexity faced by current mathematics teachers who are expected to remain at the forefront of change and deal with the consequences of the change. It is why I regards all enthusiastic mathematics teachers as super heroes and foundation workers of future change.

References

Boaler, J. (2015: Revised edition). *What's math got to do with it? How teachers and students can transform mathematics learning and inspire success*. New York: Penguin Books.

Boaler, J. (2016). *Mathematical Mindsets*. San Francisco CA: Jossey-Bass.

Doidge, N. (2008). *The brain that changes itself: Stories of personal triumph from the frontiers of brain science (Revised Edition).*Melbourne: Scribe Publications Pty Ltd.

Dweck, C.S. (2006) *Mindset: the new psychology of success*. New York: Ballantine Books.

Kanigel, R. (1991). *The Man who knew Infinity*. US: Charles Scribner's Sons.

Lakoff, G., & Nunez, R. E. (2000). *Where Mathematics comes from.* NY: Basic Books.

Leung, F. K. S. (2014). What can and should we learn from international studies of mathematics achievement? *Mathematics Education Research Journal, 26*(3), 579-605.

Skemp, R. (1976). Relational understanding and instrumental understanding. *Mathematics Teaching, 77,* 20-26.

Skemp, R. (1977). Professor Richard Skemp, interview by Michael Worboys. *Mathematics in School, 6* (2), 14-15.

Skemp, R. (1979). *Intelligence, learning and action*. Chichester: Wiley & Sons.

Skemp, R. (1986). *The psychology of learning mathematics (2nd ed).* London: Penguin Books.

Skemp, R. (1989). *Mathematics in the primary school.* London: Routledge.

Skemp, R. (1992). Bringing theory into the classroom: The school as a learning environment for teachers. In B. Southwell, B. Perry, & K. Owens (Eds.). *Space - The first and final frontier, conference proceedings, fifteenth annual conference of the mathematical education research group of Australia* (pp. 44-54). UWS Nepean, Sydney: MERGA.

Sousa, D. A. (2008). How the brain learns mathematics. Thousand Oaks, CA: Corwin Press.

Skinner, B. F. (1953). *Science and human behavior*. New York: Free Press.

Sternberg, R. J., & Davidson, J. E. (Eds.)(1995). The nature of insight. Cambridge, MA, US: The MIT Press.

Thurston, W. (1990). *Mathematical education. Notices of the American Mathematical Society, 37*(7), 844-850.

White, A. L. (2011). School mathematics teachers are super heroes. *South East Asian Mathematics Education Journal, 1*(1), 3-17.

White, A. L. (2013). Mathematics education research food for thought with flavours from Asia. *South East Asian Mathematics Education Journal, 3*(1), 55-71.

White, A. L. (2014). Juggling Mathematical Understanding. *Southeast Asian Mathematics Education Journal, 4*(1), 57-67. ISSN 2089-4716.