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Enjoying Maximising Opportunities For Learning Mathematics

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Abstract

In this keynote we will examine and participate in mathematics teaching and learning practices that contribute to strong learning focussed relationships and enjoyment of mathematics learning. Pedagogies, learning experiences, and caring teacher behaviours that include and extend beyond traditional mathematics teaching practices will be presented as examples of how academic relationships can be fostered towards all students making strong mathematics learning gains. The use of contexts that students find realistic, meaningful, and engaging will be discussed. A culturally responsive mathematics education model that encompasses cognitive, social, physical, and spiritual dimensions will be used to consider themes from the keynote session. Examples of research-practice links will be discussed.

Keywords: culturally responsive mathematics teaching, learning-focussed relationships, mathematics learning activities

Kei hopu tōu ringa ki te aka tāepa, engari kia mau ki te aka matua.
Cling to the main vine, not the loose one.

Introduction

Teaching mathematics in ways that enthuse learners, give them confidence in their mathematical ability, and assist them in seeing real world applications of their learning are all important for effective learning. Teaching in ways that cater for, and respond to, students' ways of being and knowing is important for engagement, maximising pleasure in learning, and for transfer of learning to students' own real world contexts. In this presentation we will consider how teachers can maximise students' engagement in, and passion for, mathematics learning. Reflecting on the proverb above, we will be exploring the 'main vines' of mathematics teaching and learning. To do this we will consider a range of ways to respond to students' cultures, including using pedagogies, protocols, models, and realistic and meaningful contexts to promote mathematics learning. Partnerships between teachers, teacher educators, and researchers are known to inform and promote developments in our field. I will draw from a range of research and writing projects undertaken in New Zealand that collectively involved researchers, teachers, student teachers, and school students.

Making opportunities for strong mathematical classroom investigations and discussions is important for enjoying mathematics teaching and learning. In exploring the themes of this talk we will consider how to create environments that are conducive to students taking an active part in mathematical investigations and discussions. We will explore a range of factors and mathematics learning activities that help create such environments and open up productive mathematical discussions.

Culture And Mathematics

There are many ways of thinking about 'culture'. Culture refers to everything that helps define who we are – the ways we do things, how we act, how we behave, how we interpret events and interactions and how we respond to these, how we celebrate, and how we ask and answer questions (Banks, 2006; Gay, 2010). What does responding to culture mean in mathematics teaching and learning? We are all part of many cultural groups – our professional group (teachers, researchers, teacher educators), our community groups (linked to out religions, hobbies, sports, pastimes), groups determined by where we live, and those of our heritage cultures, often linked to our ethnicity. Our students similarly are part of many cultural groups – school culture, social networks, heritage cultures, religions, sports – that impact on their ways of being and thinking as well as on what they bring with them to their learning. In this talk, I use the word 'culture' broadly to refer to everything that makes students who they are. Which aspects of culture should we attend to in our teaching? Which can we attend to? And, how can we attend to these in ways that acknowledge, empower, enhance comfort, and most importantly, maximise students' mathematics learning? Are the same or different strategies suitable for attending to culture within initial teacher education and professional development?

The international literature offers many ideas to help answer these questions. There are various factors to consider: managing effective teacher-student relationships (Bishop, Berryman, Tiakiwai & Richardson, 2003; Ladson-Billings, 1994); listening to students' perspectives about their learning (Macfarlane, 2004); and linking learning to students' experiences and interests (Gay, 2010; Kanu, 2011; Presmeg, 2007). Teacher education needs to help aspiring teachers learn to teach in culturally responsive ways and realise what knowledge they need to acquire to do so (Nieto & Bode, 2008; Villegas & Lucas, 2002). New Zealand education policy requires that teachers attend to students' cultural heritages in their teaching. There is a particularly strong focus on ensuring schools and classrooms enable Māori students to experience and enjoy academic success as Māori (e.g., Ministry of Education, 2007, 2008, 2011). In our team's research and practice we have sought to find ways to satisfy these policies and their predecessors.

There is a range of factors that contribute to creating classroom environments that attend to students' cultures in mathematics teaching and learning. We will touch on four inter-related areas: using realistic contexts; using pedagogies, protocols, and languages consistent with students' heritage cultures; attending holistically to students' needs; and ensuring effective teacher-student relationships are in place. Mathematics classroom examples will be used to illustrate each. I will use examples from Aotearoa New Zealand. Please consider these in relation to ideologies and practices suitable for the teachers, learners, and researchers you work with.

Realistic and Meaningful Contexts

We can reflect cultures in our teaching through using realistic and meaningful contexts for our learning tasks (e.g., Harvey & Averill, 2012). In New Zealand the term 'ako' means teaching and learning – the one word indicates the reciprocal nature of teaching-learning situations. Every time we teach, we learn – we use informal diagnostic assessment to work out suitable next learning steps, we find out about students' misconceptions and which concepts they find simplest and which more complicated. When we teach using realistic contexts we can also learn a lot about our students as people. Given the right classroom environment, the contexts can open up opportunities for students (and teachers) to share their knowledge, show their passions and expertise, and bring their own personality and humour to their learning.

Cultural knowledge, sensitivity, care, and respect are very important in using contextual problems as every culture has different dos and don'ts, things that make others feel comfortable or uncomfortable. It is very hard for people to learn well when they feel offended or uncomfortable. For example, internationally, food is often used to help students develop fraction concepts. However, in some New Zealand communities it can be offensive to use food as a classroom material, so it is safest for New Zealand teachers to use alternatives to food. When there are many cultural groups in a classroom it can be a big challenge for teachers to know enough about all of their students to always teach sensitively.

Connectionist learning theories (Askew, 2010) hold that students learn new ideas by being able to connect them with things they already know. Realistic mathematics education (Gravemeijer, 1994) also emphasises the importance of using realist contexts to help learning and to assist students to see the relevance and usefulness of the learning in their everyday lives (e.g., Sembiring, Hadi & Dolk, 2008). Teachers need to know their students well to use contexts safely, particularly contexts linked to heritage cultures (Gay, 2010). However, teaching using contexts can be powerful for learning about our students while promoting mathematical thinking and learning. For example, setting up graphs that use contexts students know about can help them identify personally with the concepts and share their own experiences (e.g., Figure 1). Students can be asked to identify people they know that could be represented by the points marked on the graphs, and to explain why the points represent those people well. Their answers will provide good diagnostic information about their understanding of how graphs work as well as some insights into their own experiences. A next step can be to provide axes with no labels and ask students to choose a context for the graph and labels, to include two or three points, and describe who or what the points could represent. The graphs can be made outdoors using chalk or tape on the playground with students standing inside the graph at the places that best represent them. Students can share

information they know about and bring their own ideas and humour to the activity. These things help them engage with the class, the teacher, and with mathematical ideas.

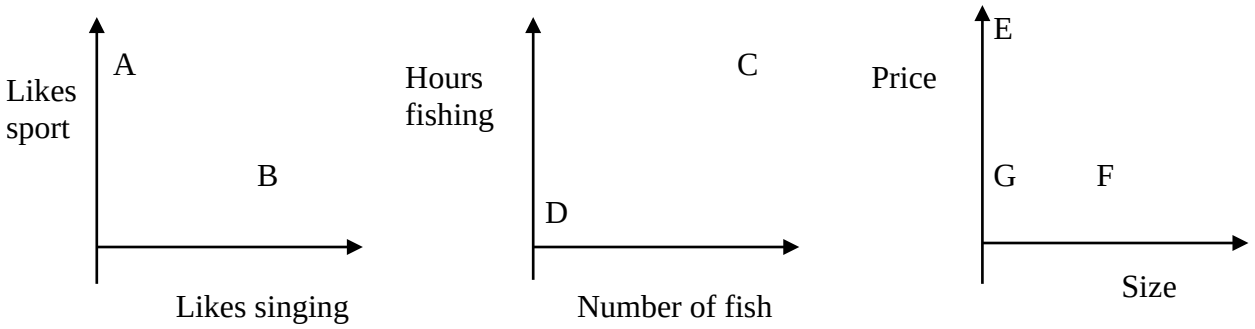

Figure 1. Realistic contexts for developing understanding of graphing

There are many ways of linking the contexts of mathematics learning activities to students' prior experiences and knowledge, while opening up opportunities for acknowledging students' cultural heritages. Some examples of realistic contexts that can open up mathematical and contextual discussions include:

- identifying geometrical features, shapes, properties, and transformations that can be found in flags of different countries (Figure 2) or simple jewellery (Figure 3); and
- collecting statistical information from students and comparing theirs with those of other groups, such as other classes in the school or, if the data are available, students nationally (e.g., Censusatschool, http://www.censusatschool.org.nz/).

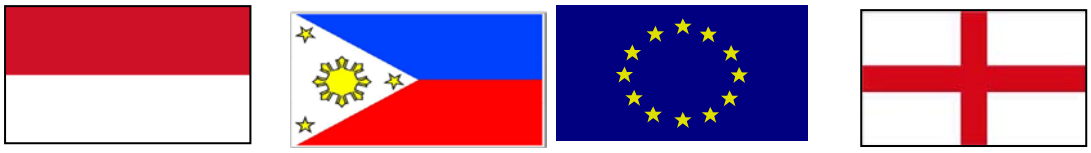

Figure 2. Flags as a context for discussing geometrical features, properties and transformations

Figure 3. Samoan jewellery as a context for geometrical transformations (photographs from Averill, Phillips and French, 2003).

Pedagogies, Protocols, and Languages Consistent With Students' Heritage Cultures

We can reflect cultures in our teaching through using pedagogies and languages consistent with those traditionally and currently used in the cultural groups of our students (Averill et al., 2009). Cultures use a range of pedagogies such as teaching using proverbs, stories, legends, songs, games, modelling, and working together/participation (e.g., Hemara, 2000). There are many ways of incorporating pedagogies and languages consistent with our students' cultures into our mathematics teaching.

This keynote opened with a *whakatauki* – a proverb – a pedagogy consistent with many cultures for setting the scene and explaining the purpose of what follows. There are many legends that we can use to introduce mathematical concepts (such as time, distance, mass, estimation, counting, patterns...), and excellent story and picture books that can provide ways to link students' learning to their reading (e.g., Perger, 2010). Songs are also a very useful tool for aiding memory and for having students share an experience that is fun, and that encourages them to breathe, think, and participate. In our teacher education lectures, we often use songs or chants to help students remember terms and rules and to set up and explore number patterns (Figures 4 and 5). We also try to know about and use protocols that enable all students to feel comfortable to participate in learning. This could be by starting the lecture with a prayer and making time to ensure we all have introduced ourselves fully and appropriately when we first meet.

1	3
1 2 1	3 5 3
1 2 3 2 1	3 5 7 5 3
1 2 3 4 3 2 1	3 5 7 9 7 5 3
1 2 3 4 5 4 3 2 1	3 5 7 9 11 9 7 5 3

Figure 4. Number Pattern Action Chant

Whanui, whanui	Luas, luas	Wide, wide
Whaiti, whaiti	Sempit, sempit	Narrow, narrow
Teitei, poto, iti	Tinggi, pendek, kecil	High, short, small
Teitei, poto, iti	Tinggi, pendek, kecil	High, short, small
Tawhito, tawhito	Kuno, kuno	Ancient, ancient

Figure 5. Action Song of Measurement Terms

Using games to develop mathematical understanding is another way of using pedagogy consistent with that used in many cultural groups. Many games have mathematical elements (e.g., Mancala) and many depend on decisions about relative likelihood (e.g., Beetle). Games are fun and engaging and can enable children to be themselves. Probability lends itself to investigating using games. Dice games are possibilities as are other games

familiar to many children, such as 'Rock, Scissors, Paper'. Children can be asked to predict outcomes, relative likelihoods, then can play the game and systematically record their results to compare the results of their game with their predictions. Theoretical probabilities can be used to analyse the games and to compare with their experimental results.

Many cultural groups use ways of modelling, working together, and mentoring – experts and more experienced people guiding those with less experience. In Aotearoa New Zealand, *tuakana-teina* is a pedagogy that is sometimes described as an older sibling mentoring their younger brother or sister. Many New Zealand teachers use some form of student-student mentoring that parallels the *tuakana-teina* concept.

We have now briefly considered mathematics teaching that uses realistic contexts and pedagogies, protocols, and language consistent with students' heritage cultures. Next we move to investigating culturally responsive classroom practice using a culturally-based holistic model of health and wellbeing. To examine the model, we will return to some of the activity examples we have discussed so far.

Attending To Our Students' Needs, the Whare Tapa Wha: A Model For Health And Wellbeing

In my thesis study, I wanted to find out how teachers make strong teacher-student academic relationships. I observed 100 lessons in three multicultural city schools to record and analyse how teachers made and kept effective relationships (Averill, 2012). I analysed the data using Durie's (1998) model of health and wellbeing. The model uses a four-sided house as a metaphor; the four sides are *te taha hinengaro* (related to thinking), *te taha whānau* (the social side), *te taha wairua* (the spiritual side), and *te taha tinana* (relating to our physical selves). My study looked at how teachers created strong teacher-student relationships in their mathematics teaching in relation to each of these four aspects of health and wellbeing. We will look at each in turn.

Teacher practices that developed cognitive aspects of health and wellbeing (*te taha hinengaro*) included teachers reinforcing firm boundaries, setting high (and attainable) expectations and making sure students were aware of these, sharing the learning purpose, and maintaining a strong focus on learning. Teachers kept students engaged by challenging their thinking, involving them in decision making, varying the lesson activities, and having a sense of urgency for completing activities. Practices such as co-constructing learning, differentiated learning, taking prior learning into account, and feeding back and feeding forward were also important for responding to students' cognitive needs.

Teachers developed social aspects of students' health and wellbeing (*te taha whānau*) through nurturing students' sense of classroom community, for example by encouraging students to be responsible for everyone's learning, themselves, and others. Teachers used inclusive language (e.g., we'll look at…, let's try...), showed they felt students' learning was

important, and incorporated activities that encouraged class community (e.g., through students working together and teachers taking part in learning tasks themselves). Teachers used activities that encouraged students' sharing of their own knowledge and personalities.

Students' emotional and spiritual health and wellbeing (*te taha wairua*) was promoted by teachers showing respect for students, providing timely assistance, and encouraging student ownership of their learning. Teachers interacted one-to-one many times with many students every lesson, set work of appropriate challenge, and provided repeated opportunities for students to feel academic success and satisfaction. Teachers attended to students' emotional and psychological needs, for example by discussing their work in quiet one-to-one discussions and using specific praise and encouragement. They were consistent, explained their decision making, sought and used students' responses to promote learning, and showed they liked their students and enjoyed working with them.

Academic teacher-student relationships were strengthened by teachers attending to students' physical wellbeing (*te taha tinana*) through ensuring the classroom was well lit, had fresh air and sufficient heat, and showing concern for their students' health. They made opportunities for students to move around inside and outside of the room to carry out learning activities, for example through students writing questions and responses on the board, working outside, with others, and with equipment. Linking mathematics learning to other curriculum areas such as physical education, drama and dance, and science can be one way of incorporating movement in mathematics lessons. For example, links with science learning are possible when measuring the horsepower needed for people moving up steps at different speeds. Horsepower can be calculated using:

$$\text{HP} = \frac{\text{mass (kg) x (height moved) (in m)}}{\text{time (seconds)}}$$

Some teachers used classroom protocols and routines that students expected and were comfortable with, such as greeting and farewelling each student, starting each lesson with a problem, game or prayer, and having consistent ways of giving feedback on work and homework. Each of these protocols contributed to several of the dimensions of wellbeing. The study indicated that all of the teacher practices mentioned above helped develop teacher-student relationships and helped students enjoy their learning, teachers enjoy their work, and for all to enjoy one another's company, all very important ways to maximise students' learning.

Now we will return to the mathematics learning activities discussed above to see how using them can impact on the four dimensions of students' health and wellbeing. Many of the activities involve mathematical purpose, some social aspects, some physical movement, and various ways for students to share of themselves in their learning (Table 1).

Table 1.
Examples of links between mathematics learning activities and the whare tapa wha.

	Taha hinengaro	*Taha whānau*	*Taha wairua*	*Taha tinana*
Number chant	Skip counting, Number patterns, algebra	All chanting together	Performance, having fun, having success	Physical movement
Graphs with contexts	Graphing, axes, plotting points, explaining features of graphs	Learning about one another	Acknowledging individuals' knowledge and experiences	Making graphs outside
Flags and jewellery	Noticing and discussing geometrical features and transformations	Learning about one another's heritage groups and nations	Acknowledging individuals' knowledge and experiences	Making own flags and jewellery to given mathematical conditions
Measurement action song	Measurement terminology	All chanting together	Performance, having fun, having success, acknowledging languages	Physical movement
Horsepower activity	Measurement, algebra, statistics	Group work, teacher taking part	Finding something out about themselves	Physical movement
Probability game	Experimental probability, predictions, modifying game to make it more fair	Group work, teacher taking part	Having fun, being successful, using a pedagogy consistent with heritage cultures	Physical movement involved with playing a hand game

Table 1 shows examples of links between the activities we have investigated in this keynote and the four dimensions of the *whare tapa wha*. How the teacher uses the activity with their own class will affect which dimensions are present and how strongly they are reflected. The activities we have used are all potentially very productive for students' mathematics learning. They are also fun. Students enjoying their work are more likely to be relaxed, comfortable, engaged, and achieving. Enjoying their work contributes to students' mathematical identity, their confidence in their mathematical ability, and their motivation to learn more mathematics. How teachers use the activities is very important in maximising how

they can contribute to mathematics lessons that attend to the four dimensions of students' health and wellbeing.

Strong Academic Teacher-Student Relationships

Strong teacher-student relationships are very important for effective teaching and learning (Averill, 2012; Gay, 2010). They are particularly important when teaching using contexts and cultural elements that are important to students. They help students feel comfortable to be themselves, ask questions, and offer their ideas. They help increase the chance that teachers will know students well enough to use suitable contexts with care and respect. Above all, respect for others, care for learning, and care for individuals are essential for developing strong teacher-student relationships. Using learning activities that include realistic and meaningful contexts, incorporate a range of pedagogies, and attend to students' academic, social, emotional, and physical needs are most likely to create strong relationships and effective learning.

Conclusion

Mathematics, teaching mathematics, and learning mathematics are all human endeavours. People enjoy having fun, being curious, being able to explore, investigate, discuss, problem solve, and, with suitable support, to work things out for themselves. We like to know the purpose of our activities and to share decisions about, and responsibility for, our learning. Traditional skills and practice type mathematics teaching and resources may not sufficiently capitalise on these human characteristics. Re-examining mathematics teaching and learning in relation to ensuring pleasure, awareness of the uses of mathematics, and confidence-building, whilst maintaining a keen focus on effective learning and achievement, is timely. Doing so in ways that maximise our own pleasure in our teaching is important for our own learning, developing our field, and developing strong learning-focussed teacher-student relationships. All of these are valuable ingredients towards maximising students' mathematics achievement. Further research is needed to investigate the human elements of teaching and learning mathematics in relation to specific learning contexts, and how these can be emphasised towards maximising students' engagement with mathematics and their achievement.

Implications of this work for teachers include teacher behaviours that promote effective relationships and that go beyond traditional teaching practices. Teachers can develop strong academic relationships by attending to the specific and holistic learning needs of their students through:

- incorporating pedagogies, protocols, and languages of the heritage ethnicities of their students

- using realistic and meaningful contexts of mathematical learning activities
- using learning tasks that involve movement and fun
- expecting and promoting strong academic progress
- prioritising one-to-one teacher-student interactions
- showing respect for students and their learning
- incorporating collaborative learning tasks
- making opportunities for sharing personal identities, and
- ensuring the learning environment is physically and emotionally comfortable.

Learning mathematics is a human activity and attending to all of these needs enables mathematics teaching and learning to be effective and enjoyed by all. In these ways we can test the local vines of mathematics teaching and learning, cling to those that are strongest and strengthen those that are loose – to make all of our students learn mathematics joyfully and well.

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