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Algebra Tiles as Physical Manipulatives to Support Indonesian Students' Understanding of Linear Equations of One Variable

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Abstract

The research aims to identify how to employ algebra tiles to support students' understanding of solving linear equations of one variable. Algebra tiles encompass square and rectangular tiles that represent numbers and variables. This research incorporates a best practice lesson series conducted at SMPN 18 Tangerang, Indonesia. The competence achievement indicators are modelling algebraic expressions by administering algebra tiles, solving linear equations of one variable employing algebra tiles, and solving linear equations of one variable without utilizing algebra tiles. This study uses a teacher-as-researcher methodology. Data was collected from photos, videos, worksheets, and students' works. To simplify the form of algebra tiles, reducing and balancing ways were administered to the students. The objective of lowering or balancing both sides of linear equations of one variable is to obtain the square tiles equal to one rectangular tile. In solving linear equation of one variable, algebra tiles support students in enhancing their understanding.

Keywords: Algebra tiles, linear equations, one variable, physical manipulatives, teacher-as-researcher

Introduction

Students are required to possess a solid understanding of algebra to solve linear equations with one variable. According to Breggren (2015), algebra is a branch of mathematics that applies arithmetic operations and formal manipulation as abstract symbols understood as variables rather than specific numbers. All subsequent mathematics, statistics, and other allied fields, including the natural sciences, computer science, economics, and business, are built based on algebra. Beginning algebra is a topic taught in the seventh grade of secondary school and has been referred to as the previous mathematics topic (Cai et al., 2005). Al Khawarizmi's text explained that the goal of studying algebra is to solve equations (Krantz, 2010). Furthermore, linear equation of one variable is also necessary for learning other topics in mathematics.

Solving linear equation is an essential process in algebra. However, it confuses most students (Magruder, 2012). In comprehending the concepts of linear equations, one of the common mistakes is by implementing arithmetic operations (Jupri, Drijvers, & Van den Heuvel-Panhuizen, 2014). One of the examples is to identify the value of x in the equation $2x = 7$, then x requires 7 divided by 2, but the students generally answer with $x = 7 - 2$.

Based on the researcher's interview with students at SMPN 18 Tangerang, most frequently struggle to solve problems associated with algebraic expressions. They are confused with solving equations which embody variables. However, students are demanded to understand linear equations of one variable. To help students solve the linear equations of one variable, a teacher may employ a learning media in the teaching process. Kasim, Rochaminah, and Hadjar

(2016) signified that the ability of students to understand concepts and solve problems correlated with linear equations of one variable is still very low. They researched it and discovered that media or teaching aids could enhance students' learning outcomes of the linear equations of one variable at class VII MTsN. Poso Pesisir. Employing a learning media is expected to provide meaningful mathematics learning for students. It aims to enhance students to obtain a better understanding of solving the linear equations of one variable concept.

Manipulatives

Manipulatives in mathematics learning are various concrete materials or objects that can be moved or touched and employed to support students in learning mathematics. Manipulatives encompass all physical objects administered to represent abstract concepts. Manipulatives assist students in physically manipulating and acting out abstract concepts of mathematics, like algorithms. There are numerous kinds of manipulatives, for instance, counters, pattern blocks, base ten blocks, calculators and even students' own fingers (Mink, 2012).

Salmah (2019) asserted that two types of manipulatives are physical and virtual manipulatives. Teachers employ manipulatives to enhance abstract reasoning and interact with mathematics concepts concretely. It is demanded that mathematical concepts are conveyed, not merely symbolically. Moreover, manipulatives allow students to reveal mathematical concepts independently and make it easier to retrieve knowledge later. The utilization of manipulatives in mathematics learning is tremendously beneficial for students, which begins with providing concrete materials to comprehend the concept before generating abstract connections (Morsidi & Shahrill, 2015; Goh et al., 2017).

Manipulatives also have the potential to teach challenging concepts like algebra (Badaruddin et al., 2018). Algebra tile is one of them. The manipulative, characterized as an algebra tile, represent algebraic expressions, incorporating variables and constants. Students use algebra tiles to visually represent equations before solving them. (Mink, 2012). According to Kennard (2019), Algebraic tiles can help pupils with various degrees of proficiency and confidence. Students depict mathematics by "playing" with the algebra tiles to gain a more profound knowledge of algebraic expressions and formal algorithms.

Larbi and Okyere (2014), in their study about the use of algebra tiles as manipulatives and the relation with gender differences, uncovered that algebra tiles enhance meaningful learning and students' conceptual understanding. Moreover, students who explore with algebra tiles own a positive attitude toward implementing manipulatives as they can comprehend and internalize the up-to-date taught mathematical concept. Interestingly, implementing manipulatives helps close the mathematics performance gap between male and female students.

However, the limitations posed by a lack of teacher training, student apprehension when applying the new models, and students' pre-knowledge on how to solve problems through another method are all points of contention in the other research studies which criticize concrete models. Uttal, Scudder, and DeLoache (1997) argued that "even if the students were able to solve mathematics problems by employing manipulatives, they frequently fail to connect this knowledge to more traditional forms of mathematical expression" (p. 45). Furthermore, this necessitated double the work on the students learning the method to implement the manipulatives to solve problems and then learning the way to implement the symbols to complete a similar thing. However, Uttal and colleagues stated that the best manipulatives might be merely applied for teaching mathematics. In considering this criticism, algebraic tiles are implemented and designed for mathematics teaching and learning.

Algebra Tiles

Algebra tiles are square and rectangle-shaped tiles representing numbers and variables. Algebra tiles comprise three different-sized pieces. From Figure 1, the smallest tile is a square shape representing ± 1, another is a rectangular shape signifying ± *x*, and the largest is a large square shape representing ± x^2. The pieces are frequently color-coded; hence, one color represents positive values, and the other color represents negative values.

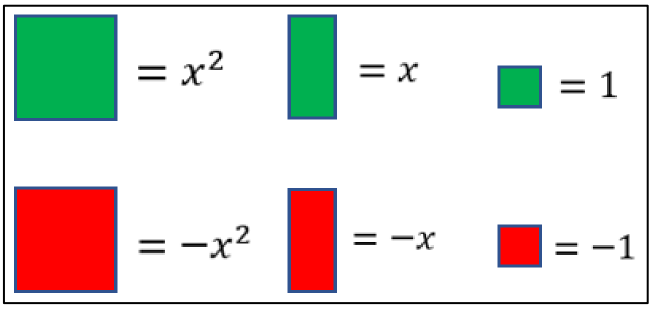

Figure 1. Algebra tiles

The add opposites attribute is assumed by algebra tiles, which means that two pieces of the same area but with opposing signs (colors) balance each other out to produce a sum of 0 (Beckmann, Thompson & Rubenstein, 2010). Mathematical tools called algebra tiles combine a geometric and algebraic approach to studying algebraic ideas. Students can better understand algebraic thinking strategies and abstract concepts by employing algebra tiles. Additionally, they give students an additional chance to resolve the algebraic problem.

Linear Equations of One Variable

An equation is a mathematical sentence in which the equal sign's left and right sides of the equal sign represent the same amount. The equal sign indicates that the amounts on either side of it are of equivalent value by acting as the fulcrum on a seesaw or as a balance. Therefore, for the equation to remain balanced, any operations performed on one side must also be performed on the other (Beckmann et al., 2010).

According to Corry (2019), equations are likely the most fundamental idea in mathematics. x+2=5 is a basic example of a formal assertion that states that a mathematical expression on two distinct sides is equal. Addition, division, finding roots, and other operations can be administered on both sides of an equation to solve it.

The standard form of a linear equation of one variable *x* can be written $ax + b = 0$, in which a and b are real numbers with $a \neq 0$. Since the variable in a linear equation possesses an implied exponent of 1, it is also identified as a first-degree equation. Due to some challenges encountered by the students on the topic of algebraic expression, the teacher enhanced a learning activity employing algebra tiles. By the end of this lesson, students will be able to: model the algebraic expressions by implementing algebra tiles; solving the linear equations of one variable by administering mathematical operations of addition, subtraction, multiplication, or division using algebra tiles; solve the linear equations of one variable by implementing mathematical operations such as addition, subtraction, multiplication, or division without employing algebra tiles. This paper examines the research question as follows: How can the algebra tiles support Indonesian students' understanding of solving linear equations of one variable?

Methods

This paper utilized a teacher-as-researcher methodology. The concept of teacher-as-researcher encourages teachers to collaborate in revising their instruction and owns its roots in action research (Kemmis & McTaggart, 1982). The researcher used the practice of learning as the finest experience in mathematics learning in the context of one-variable linear equations. The notion was a novel method of teaching and learning mathematics in the researcher's classroom, doing this best practice distinctive, inventive, and innovative research. Applying algebra tiles to enhance students' knowledge of linear equations of one variable was a novel and unusual method from other mathematics courses that aimed to strengthen the quality of mathematics teaching and learning process.

This research was a best practice process performed at grade VII.7 SMPN 18 Tangerang. It encompassed 36 students and was implemented following the schedule of the researcher's mathematics grade VII.7 SMPN 18 Tangerang on Tuesday, 26 November 2019. The researcher prepared all of the learning equipment to conduct the implementation well. It comprised a lesson plan, a worksheet, an instrument of assessment and algebra tiles.

The competence achievement indicators administered in this research were modelling algebraic expressions by implementing algebra tiles; solving linear equations of one variable by applying mathematical operations of addition, subtraction, multiplication, or division employing algebra tiles; solving linear equations of one variable by implementing mathematical operations of addition, subtraction, multiplication, or division without use algebra tiles. The data collection method used photos, videos, worksheets, and students' work.

Strategy and Implementation

The worksheet was divided into three parts following the three competence achievement indicators. In the first part, the teacher provided five different algebraic expressions and students were expected to be able to model them by employing algebra tiles. There were three algebraic expressions: $4x + 3$, $5a + 10$, $x^2 - 3x + 5$ and two linear equations of one variable: $m + 7 = -6$ and $3z + 5 = 14$.

The students discussed in groups and did the task following the first worksheet (refer to Figure 2). They had to use algebra tiles to model algebraic expressions. A few students understood it well and practiced it enthusiastically. Several were still confused about utilizing algebra tiles to model algebraic expressions. Thus, the teacher went around, observed the students as they worked, guided them, and assisted them with difficulties. The teacher continually reminded the students to work collaboratively; thus, if a student understood, he or she would guide their friends in the group. After working on a few questions, it was easier for students to model the algebra expressions by implementing algebra tiles. Then, a class discussion was organized to present the students' work results. One group demonstrated their work, and the other groups responded by asking questions or providing different solutions.

Figure 2. Students working in groups

The learning process continued by working on the second part of the worksheet. One variable has eight linear equations: $x + 3 = 8$, $9 = m + 1$, $10 = n - 4$, $-5 + x = -1$, $3m = 24$, $2x + 6 = 12$, $3z + 5 = -16$, $5x + 8 = 2x - 7$. The teacher informed the students about a zero pair (see Figure 3). A zero pair is a negative and positive unit tile which together generate a sum of zero.

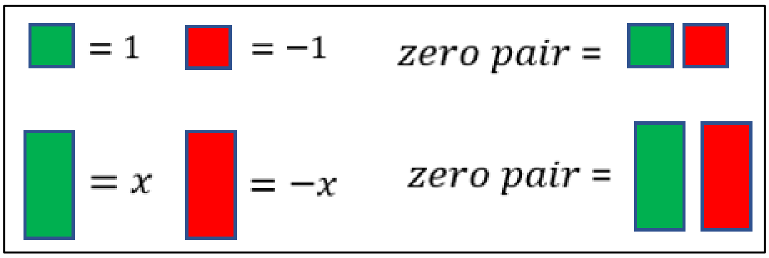

Figure 3. Zero pairs of algebra tiles

As shown in Figure 4, the students worked in their groups enthusiastically. The teacher assisted the students in solving linear equations of one variable by employing algebra tiles and observing good ideas from the students to invite them to present during the class discussion. Then, the teacher guided them to discuss their results.

Figure 4. Students working enthusiastically

Following the third part of the worksheet, the students solved by applying mathematical operations of addition, subtraction, multiplication, or division without using algebra tiles. They worked in groups to complete eight linear equations of one variable: $4 + x = 10$, $11 = n - 2$, $a - 7 = 20$, $-2 = -m + 5$, $4x = 12$, $2y + 4 = 0$, $14 = 2a + 4$, $6n - 1 = 2n + 19$

Students solved the problems more quickly than expected by performing algebra tiles (Figure 5). As they solved the linear equations of one variable by applying algebra tiles, they understood how to solve it without them. All groups were allowed to visit each other during the class discussion to exchange their solutions. After the class discussion, the students concluded the result of the group work by explaining their strategy for solving the linear equations of one variable.

Figure 5. Solving linear equations of one variable without the use of algebra tiles

Results and Discussion

The analysis indicates that students solved the linear equations of one variable more easily and quickly by implementing algebra tiles. This result follows the findings of previous studies, which indicated that in the experimental group which received intervention by employing algebra tiles, a statistically significant improvement in students was acquired (Akpalu, Adaboh & Boateng, 2018). As a result, the algebra tiles are valuable manipulatives which enhance students learning of algebra. The administration of algebra tiles has supported students in learning more about algebra. Students could efficiently respond to challenging questions, which escalated their enthusiasm for learning mathematics. Furthermore, students can cultivate a more positive attitude toward equations and mathematics (Akpalu, Adaboh & Boateng, 2018).

Van de Walle, Karp and Williams (2010) asserted that balancing is one method for completing the equations. The concept of "balancing" or" reduction" becomes fundamental in solving linear equations. Bernardz, Kieran, & Lee (1996) elaborated that it is also associated with the first three of Euclid's *Five Common Notions*. The three ideas are as follows: 1) Thing that is equivalent to something else is also equal; 2) The sums of equals, when added together, are equal; and 3) The remainders are equal if equals are subtracted from equals. These ideas become the cornerstone of how to solve one-variable linear equations. The researcher advised the students to implement the idea of addition, subtraction, multiplication, and division on both sides of linear equations.

In this study, firstly, students counted the number of algebra tiles to model algebraic expressions. They implemented the algebra tiles in representing the quantities of a provided algebraic expression. The competence achievement indicator was modelling algebraic expressions by employing algebra tiles. The students working on the algebra tiles of the provided problem are presented in this Figure 6.

Figure 6. Modelling algebraic expressions using algebra tiles.

Based on Figure 6, for the algebraic expression $4x + 3$, they organize four green rectangular tiles and three yellow square tiles. For the algebraic expression$x^2 - 3x + 5$, students administered one of the big blue square tiles, three red rectangular tiles and five square tiles. The algebraic expression $m + 7 = -6$ represented by one light green rectangular tile, seven yellow square tiles and six red square tiles. Moreover, for the algebraic expression $5a + 10$, students administered five green rectangular tiles and ten yellow square tiles. It can be perceived that students were able to model the algebraic expressions by employing the algebra tiles correctly. Per Beckmann, Thompson and Rubenstein (2010), algebra tiles can be used in simplifying algebraic expressions.

Furthermore, algebra tiles can assist students in building a conceptual understanding of a procedure, such as solving linear equations (Beckmann, Thompson & Rubenstein, 2010). Bush, Karp and Dougherty (2021) presented that algebra tiles allow students to model algebraic expressions or equations as quantities, generate equal terms, factorize, or solve equations. Moreover, in the guidebook for exploring algebra, students can model algebraic expressions by administering algebra tiles. Students are free to select the color of algebra tiles and the shape of the tiles. The objective is that students can implement manipulatives to model the provided algebra expressions (Dougherty, Matsumoto, & Zenigami, 2003).

The second competence achievement indicator is solving linear equations of one variable by implementing mathematical operations of addition, subtraction, multiplication, or division using algebra tiles. In this activity, students are required to identify the number of square tiles equal to a rectangular tile.

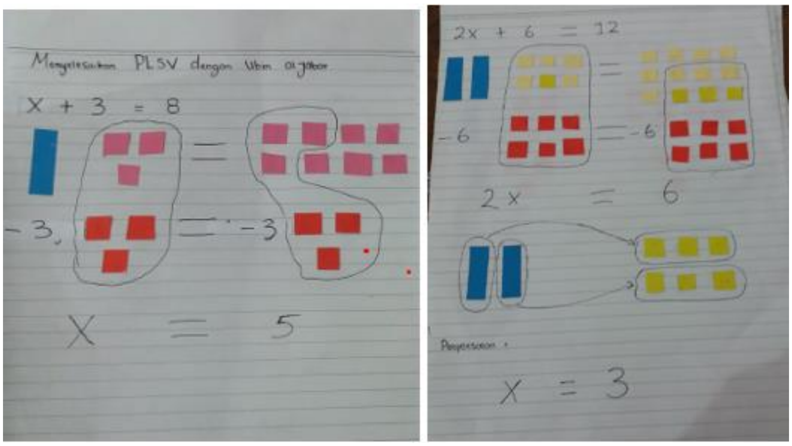

Figure 7. Solving linear equations of one variable by employing algebra tiles

In Figure 7, for the first linear equations of one variable $x + 3 = 8$, students subtracted three on both sides, represented by three red tiles. Then, on the left side, there were three pink tiles (positive) and three red tiles (negative), which resulted in a zero pair. Finally, the result on the left side is one blue rectangular tile that represents x. On the right side were eight pink tiles (positive) and three red tiles (negative). Hence, the result was five pink tiles (positive) representing the constant 5. Finally, the linear equation solution for one variable $x + 3 = 8$ is $x = 5$.

For the second linear equation of one variable $2x + 6 = 12$ in Figure 7, the students solved it by subtracting six on both sides to embody six red tiles on each side. On the left side, they formulated a zero pair that came from six yellow square tiles (positive) subtracted by six red square tiles (negative). Then, on the left side, there were merely two blue rectangular tiles representing $2x$. On the right side, twelve yellow square tiles were subtracted by six red square tiles. Thus, there were six yellow square tiles which represented the constant 6. For the next step, students divided the equation by two on both sides. They discovered that three yellow square tiles were equal to a blue rectangular tile, which was $x = 3$.

In this activity, two groups solved linear equations of one variable in the wrong way. The teacher frequently reminded them to discuss it to identify the correct answer. Finally, all students could solve the linear equations of one variable correctly.

Dialogue 1: The linear equations of one variable $2x + 6 = 12$

Hikmah : There are two blue rectangular tiles and six yellow square tiles on the left

Researcher : Yes…

Eva : 12 yellow square tiles on the right. Oh, I see…

Hikmah : We have to eliminate the six yellow square tiles on the left so that only the blue rectangular tiles remain.

Andika : So, we have to subtract by six red square tiles.

Researcher : What do the red square tiles mean?

Andika : I remember that the red square tiles represent negative or minus.

Eva : It means by putting the red square tiles, we subtract…

Hikmah : Yes, and we should do the same method on the right side.

Researcher : Do it
Eva : Ok, I will put the red square tiles on both sides.
Andika : The six yellow and six red square tiles are zero…
Eva, Hikmah : We find $2x = 6$
Andika : Now, we subtract by one of the blue rectangular tiles.
Hikmah : No, we can't do it.
Eva : Subtract by the blue rectangular tiles. Why can we not do this?
Hikmah : It will cause the right side to increase even more.
Researcher : Why did you subtract by the blue rectangular tiles?
Hikmah : Oh no, not subtract. We have to divide it.
Researcher : How did you divide it?
Hikmah : Divided by two.
Eva : Oh yes, each of them, the left and right.
Researcher : Each of them is divided into two. So, one blue rectangular is equal to?
Hikmah, Eva : Equals to 3 yellow square tiles. Yes…

Following the dialogue, it can be perceived that the students employed reducing and balancing ways to identify the simple form of algebra tiles. On both sides of the one-variable linear equations, they were in balance. The balancing method is a critical concept in locating the answer to one-variable linear equations. Both sides must be reduced or balanced to attain the number of square tiles that equals one blue rectangle tile. Some students just circled one part of the blue rectangle tile, dragged it to the three yellow square tiles, and repeated the process for the other half as they believed that one part of the blue rectangular tile equals three yellow square tiles. This method is equivalent to dividing each pair of blue rectangular tiles and yellow square tiles by two.

For the third activity, the competence achievement indicator is solving linear equations of one variable by applying mathematical operations of addition, subtraction, multiplication or division without employing algebra tiles. Students ought to solve the linear equations of one variable without algebra tiles (refer to Figure 8). They can solve it since they already understand how to solve it by performing the algebra tiles.

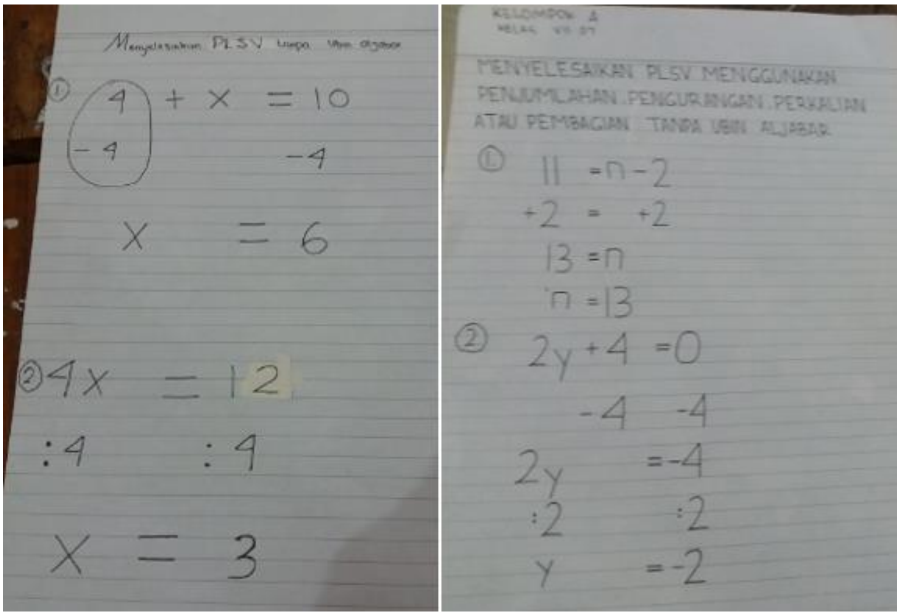

Figure 8. Solving linear equations of one variable without algebra tiles

For the linear equations of one variable $4 + x = 10$, students immediately answered it by subtracting both sides by four, and they revealed $x = 6$. They solved the linear equations of one variable $4x = 12$ by dividing both sides by four, and the solution is $x = 3$. For $11 = n - 2$, students added 2 to both sides and then revealed $13 = n$ or $n = 13$. Students solved $2y + 4 = 0$ by subtracting 4 and identified $2y = -4$. They continued by dividing by two into both sides, and the solution is $y = -2$. To determine the student's understanding, the teacher applied the exercises: $x + 6 = 10$, $8 = a - 1$, $4x + 4 = 0$ and $3 + 2m = 15$. They worked individually.

Dialogue 2: The linear equations of one variable $11 = n - 2$

Researcher : How did you solve it?
Nanda : I try, $11 = n - 2$, I have to eliminate -2
Researcher : What steps did you take?
Nanda : Subtracted by two
Researcher : Are you sure?
Nanda : Oh, I know, added by two, so we find a zero pair
Researcher : And then?
Nanda : On the left, we find $11 + 2$ and only n on the right
Researcher : What did you mean by n?
Nanda : n is the rectangular tile, and $11 + 2$ or 13 are the square tiles
Researcher : So, just added by 2 …
Nanda : I finish it, $n = 13$, wow… it's very simple

The dialogue demonstrates that students solved the linear equations of one variable by adding a particular amount on both sides so that they discovered a zero pair to isolate the variable. Students revealed various methods to solve the linear equations of one variable without employing algebra tiles. By implementing algebra tiles, students can distinguish algebraic expressions that frequently confuse them, such as”$2x$” and”$2 + x$” (Salifu, 2022). Furthermore, Sibbald in Salifu (2022) also unveiled that algebraic tiles are the best method for teachers to instruct algebraic expressions; for instance, the principle of neutralizing or making “zero” can be performed easily by implementing negative tiles and positive tiles. It is beneficial for students to solve the linear equations of one variable.

The Impact of Using Algebra Tiles in Mathematics Learning Process

At the end of the learning process, the teacher led the students to reflect on the learning process. They elaborated that they were pleased to learn mathematics by utilizing algebra tiles. It assisted them in solving the linear equations of one variable efficiently. Most students were more active and interested in practicing directly solving the linear equations by employing algebra tiles (shown in Figure 9). Students reflected that the learning time allocation was insufficient as they were enthusiastic about using algebra tiles in the learning process.

In general, the learning experience was successful, fun, and purposeful. The students’ ability to solve one-variable linear equations is what matters most. This effect was consistent with Salifu (2022), which discovered that algebra tiles were useful manipulatives for teaching students how to solve linear equations with one variable, understand challenging mathematical ideas, and deal with related problems in everyday life.

Figure 9. Students happily learn mathematics by employing algebra tiles

Conclusion and Recommendation

In conclusion, algebra tiles help students model algebraic expressions and solve linear equations of one variable. Algebra tiles make it easier for students to solve linear equations and enhance their understanding. The implementation of algebra tiles produced excitement, and the students were more active and motivated to participate in the learning process. Mathematics became an engaging, joyful, and meaningful course for students. When teaching mathematics with manipulatives, careful planning is essential. The entire class discussion demanded extra control during the learning process.

This study cannot claim that its findings may be implemented in other classes or schools due to the limitation of the sample and the data gathering. It is evident that the teacher-as-passionate researchers would affect the class and the impact of "something new". There are techniques to prevent these potential contaminants, but more research is required to determine how effective this strategy is. However, just one learning cycle is presented in this work; the researcher will assess every step before implementing a second cycle. However, it is expected that by reading this paper, other teachers will be inspired to implement it in their classrooms. Meanwhile, although the demands of teaching are heavy, teachers are encouraged to be more creative and innovative in utilizing mathematics learning media. Additionally, teachers will require the support of their school principal in producing innovative teaching.

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